Passion over Reason?

Mixed Motives and the Optimal Size of Voting Bodies*

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Abstract

We study a Condorcet jury model where voters are driven by instrumental as well as expressive motives. We show that arbitrarily small amounts of expressive motives significantly affect equilibrium behavior and the optimal size of voting bodies. Increasing the size of voting bodies always reduces accuracy over some region. Unless conflict between expressive and instrumental preferences is very low, information does not aggregate in the limit. In that case, large voting bodies are no better than a coin flip at selecting the correct outcome. Thus, even when adding informed voters is costless, smaller voting bodies often produce better outcomes.

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1 Introduction

Why do people vote? Obviously, people vote to influence the outcome of an election. Yet these "instrumental" motives (as they are known in the literature) are unlikely to be the sole reason. Voters influence the election outcome when they are pivotal and this occurs only rarely, even with a modest number of voters. The fact that many people nonetheless bother to vote suggests that they also derive some consumption utility—known as "expressive" motives—from the act of voting itself, independent of the outcome of the election.

The distinction between expressive and instrumental motives is immaterial from a modelling perspective if (i) expressive utility only depends on whether you vote and not on how you vote—this is the situation described by Riker and Ordeshook's famous 'D' (duty) term—or (ii) expressive motives always coincide with instrumental motives. In either case, the standard assumption that voters only have instrumental preferences is harmless because, conditional on showing up, voters behave *as if* they only had instrumental preferences.

As argued by Fiorina (1976) and others, usually, people's expressive utility not only depends on whether they vote, but also on how they vote. This is most apparent under an open ballot, where your vote may affect how other people perceive you. The secret ballot shelters a voter from the perceptions of others, but does not insulate him from self-perceptions. Therefore, expressive utility may also depend on whether a vote is consistent with personal norms, values, and identity. While the effect of these intrinsic expressive motives might be small, for most voters, they are arguably not zero.[1] In addition, instrumental and expressive payoffs may not always be perfectly aligned. This is obvious in legislative settings, where representatives may be torn between casting a vote that is "right" from a public policy perspective versus casting an opposing vote that will play better with constituents back home.

[1] A growing body of evidence suggests that intrinsic motives to behave in line with aspirational views of oneself are, in fact, not that small. See, for instance, Gneezy and Rustichini (2000) and others. Akerlof and Kranton (2010) offer a rich account of the connection between self-perception and behavior.

But conflict may arise even at the level of intrinsic expressive motives. For instance, a union member may see little hope that a strike will succeed, but view voting against the strike as conflicting with his self-perception as a strong union supporter.

We show that the resolution of tension between instrumental and expressive motives crucially depends on the size of the voting body. In small voting bodies, there is a reasonable chance that a single vote affects the election outcome and, hence, instrumental motives dominate. In large voting bodies, the chance of being decisive is negligible and expressive motives come to the fore. Finally, for bodies of intermediate size, instrumental and expressive motives jointly determine voting behavior.

The idea that the size of a voting body can affect voting behavior has a long history in political economy. In Federalist No. 58, James Madison pointedly observed that "the more numerous an assembly may be, of whatever characters composed, the greater is known to be the ascendancy of passion over reason." To the extent that instrumental motives reflect "reason" and expressive motives reflect "passion," our result represents a formalization of Madison's intuition: as voting bodies grow in size, passions (i.e., expressive motives) become dominant. While Madison viewed this outcome as something to be guarded against, as long as passion and reason are correlated, we show that "passionate" voting need not produce poor policies.

Returning to the example of the union voting whether to go on strike, what are the chances that it will take the right decision (i.e., go on strike if and only if the strike will be successful and management will back down)? How does it depend on the size of the union, and how important is the prevalence of conflict between instrumental and expressive preferences? To study these questions, we amend the canonical Condorcet jury model and incorporate expressive preferences into the voting calculus. We are agnostic about the degree to which expressive motives affect overall payoffs. They may be a small component or a large one. However, even when expressive motives constitute only a small share of payoffs, we show that they dominate all other considerations in large elections. Put differently, when voters have any expressive preferences at all, the reduced-form model of purely instrumental payoffs produces misleading results. Indeed, our main contribution is to show that many results from standard voting models are overturned, or in need of amendment, when expressive motives are also a consideration, no matter how small.

A key variable, what we term *malleability*, is the degree to which expressive preferences are influenced by the "facts," i.e., information as to what is instrumentally the better option. For example, some union members may view voting for an unsuccessful strike as silly and wrong. Others may be more rigid and unwilling (or unable) to put aside their strong norms about supporting the union leadership, regardless of the chances of success. We show that, if expressive preferences are sufficiently malleable, purely expressive voting can still produce good outcomes: the Condorcet jury theorem holds and large voting bodies take the correct decision with probability one, despite the fact that no one is voting instrumentally. When expressive preferences (or "norms") are relatively impervious to facts, however, large voting bodies do no better than a coin flip.

This might seem to imply that voting bodies should be as large as possible when expressive motives are malleable (or, in a political context, when "independents" make up a large fraction of the voting body), and small when they are not. In fact, this would be a mistake. Even when expressive motives are malleable such that, in the limit, large voting bodies take the right decisions, we show that there always is a (potentially large) region where increasing the size of the voting body leads to worse decisions. That is, for intermediate-sized voting bodies, the informational gains from adding more voters can be swamped by informational losses from more expressive voting.

The remainder of the paper proceeds as follows. We first place our findings in the context of the extant literature. Section 2 then sketches the model. Section 3 characterizes pure strategy equilibria, while section 4 provides a complete equilibrium characterization.

Section 5 studies the quality of decision making as the size of the voting body grows. Finally, Section 6 concludes. All proofs are relegated to an Appendix.

Related Literature The idea that voters must be motivated by considerations other than the purely instrumental dates back to, at least, Downs (1957). Subsequently, many authors have proposed adding expressive motives to the voting calculus, primarily as a way of explaining turnout. While Riker and Ordeshook (1968) is an early version of this idea, Fiorina (1976), Brennan and Buchanan (1984), Brennan and Lomasky (1993), Feddersen and Sandroni (2006), and others have also offered models along these lines. The rationales for expressive voting have varied across models. They include duty, identity, norms, and various other considerations. (See Hamlin and Jennings, 2006, for a survey.) Our model is in the same spirit. However, we can be agnostic about the exact rationale for expressive voting. That is, even though we shall couch expressive motives in terms of norms, the particular source driving expressive payoffs is of little consequence for our analysis. All that matters is that voters derive some consumption utility from voting in a particular way, and that this utility is less than perfectly correlated with their instrumental utility.

Coate, Conlin, and Moro (2008) and Coate and Conlin (2004) present empirical evidence for the importance of non-instrumental considerations in voting. There is also a growing experimental literature on expressive voting. While expressive motives received only weak support in early studies (see, for instance, Tyran, 2004), more recent studies offer significant evidence. For instance, Feddersen, Gailmard, and Sandroni (2009) find that ethical expressive voters drive turnout and policy outcomes in large elections. Interestingly, Shayo and Harel (2011) find that instrumental and expressive voting depend critically on the probability that a voter is pivotal—more expressive voting is observed when voters are less likely to be decisive.

Our work differs from earlier studies in at least two dimensions. First, building upon the observation that voting becomes more expressive when the chance of being pivotal falls, we study the implications for the optimal size of voting bodies. Second, the central driver of our results is the prevalence of conflict between instrumental and expressive motives, a notion that is absent from the extant literature.

Our paper also contributes to the vast literature on information aggregation in voting. The polar case of our model without expressive motives is a special case of Feddersen and Pesendorfer (1998). That paper, as well as Feddersen and Pesendorfer (1997), shows that a version of the Condorcet jury theorem holds very generally—large elections succeed in aggregating information.[2] However, these papers assume that preferences are purely instrumental. Our main finding is that many of the results in this literature are overturned, or require amendment, when one adds even arbitrarily small amounts of expressiveness to voter preferences.

Finally, our concern with the optimal size of voting bodies connects to the literature on the optimal design of committees (see, e.g., Mukhopadhaya, 2003, Persico, 2004, and references therein). This literature highlights free rider problems in information acquisition—the larger the committee, the less the incentives for an individual voter to become informed—which places a check on the optimal size of voting bodies. We offer a different rationale for limiting committee size, which is relevant even when informational free riding is not a problem.

2 Model

We study a simple model of voting, where voters are driven by instrumental as well as expressive motives. Suppose there are two equally likely states, labeled $\theta \in \{\alpha, \beta\}$, and a simple-majority vote with two possible outcomes, $o \in \{A, B\}$. Each of $n+1$ voters, where n is even, receives a conditionally independent signal $s \in \{a, b\}$. With probability $r \in \left(\frac{1}{2}, 1\right)$

[2] For similar results see, e.g., McLennan (1998), Fey (2003), and Myerson (1998). On the other hand, Bhattacharya (2008) offers a negative result. He analyzes a class of instrumental models where information does not aggregate. Goeree and Yariv (2009) offer experimental findings consistent with Condorcet jury theory.

a voter receives a "true" signal—that is, an a signal when the state is α and a b signal when the state is β. Otherwise, the voter receives a "false" signal, defined in analogous fashion.

A voter's payoffs are determined by the outcome of the vote, o, the underlying state, θ, and his individual vote v, $v \in \{A, B\}$. Outcome A is objectively better in state α, while outcome B is better in state β. Specifically, all voters receive a payoff of 1 if the better outcome is selected and a payoff of 0 if the worse outcome is selected. We shall refer to this aspect of a voter's payoffs as his instrumental payoffs. In addition, voters also derive direct consumption utility from voting in a particular way. We shall refer to this aspect of a voter's payoffs as his expressive payoffs. Expressive payoffs may be intrinsic, i.e., derive from how voting a certain way affects one's self-image, or they may be extrinsic, e.g., a representative may have to explain his vote to constituents back home. We can be agnostic about the precise rationale for these expressive payoffs. What matters is that voters derive some consumption utility from voting in a particular way and that this utility is less than perfectly correlated with their instrumental payoffs. However, for concreteness, we shall couch expressive motives in terms of norms: voting in a fashion consistent with one's norms yields an expressive payoff of 1, while casting a vote against one's norms yields an expressive payoff of zero. Finally, let $\varepsilon \in [0, 1]$ denote the relative weight a voter places on expressive payoffs, while complementary weight is placed on instrumental payoffs.[3]

Next, we turn to how expressive preferences, or norms, are determined. Suppose that, ex ante, norms are such that, with probability $\rho \geq \frac{1}{2}$ and independently across voters, a given voter views voting for A as normative.[4] After the state has been realized and the voter has received his signal, his expressive preferences might change. Specifically, we suppose that with probability $q \in [0, 1)$ and independently across voters, a voter is influenced by his signal and adopts a norm consistent with his posterior beliefs about which outcome is more likely to be superior. Thus, with probability q, a voter receiving an a signal adopts voting for A

[3] Our payoff specification accomodates any preferences of the form $U = \delta_o \cdot I_{\{o \text{ is correct}\}} + \delta_t \cdot I_{\{v=t\}}$, for arbitrary $\delta_o, \delta_t > 0$. Here, I denotes the indicator function.

[4] Assuming $\rho \geq \frac{1}{2}$ is without loss of generality. For the opposite case, simply relabel the outcomes.

as the norm while, with the same probability, a voter receiving a b signal adopts voting for B as the norm. With the complementary probability, $1-q$, the voter sticks to his ex ante norm. One can think of q as representing how malleable expressive preferences (or norms) are to facts.

In environments where party affiliation plays an important role, one can think of voters with malleable norms as independents and voters with rigid norms as partisans. In this interpretation, partisans, who make up a fraction $1-q$ of the population, receive an expressive payoff from voting for "their" candidate. The remaining fraction, q, are independents. They receive an expressive payoff from voting for whomever they believe is, instrumentally, the better candidate.

Formally, a voter's norm is summarized by his type t, $t \in \{A, B\}$. An A type receives an expressive payoff from voting for A, while a B type receives an expressive payoff from voting for B. With probability q and independently across voters, a voter's type is determined by his signal; that is, an a signal induces type A, while a b signal induces type B. With probability $1-q$ a voter's type is not influenced by his signal, such that his type and signal are uncorrelated. In that case, the voter's type is A with probability ρ.

To summarize, a voter with type t who casts a ballot v in a vote that produces outcome o in state θ receives payoffs

$$U = \begin{cases} 1 & \text{if} \quad o \text{ is correct and } v = t \\ (1-\varepsilon) & \text{if} \quad o \text{ is correct and } v \neq t \\ \varepsilon & \text{if} \quad o \text{ is incorrect and } v = t \\ 0 & \text{if} \quad o \text{ is incorrect and } v \neq t \end{cases}$$

To fix ideas, consider a union voting on whether to go on strike. Individual union members have information as to the likelihood that the strike will be successful and management will back down. Each member also has norms as to how one "should" vote. Norms may be formed by solidarity with fathers and grandfathers who also worked for the union. They may be influenced by social factors: How can I look my co-workers in the eye if I vote a certain way? Norms may be formed by ideology, by a sense of justice about labor-management power relations, or a host of other factors. When voting norms are in line with the facts on the ground—for example, I think the strike will succeed and my norms say to vote for a strike—the voting calculus is simple. Tensions arise when the two collide. A union member may see little hope that the strike will succeed, but feel that the governing norm is to support the union leadership and vote for a strike. The model tries to capture the idea that, for some voters, norms and passions are malleable depending on the facts of the case, while for others they are not. In the end, a voter's payoff is determined both by instrumental factors—whether the strike is successful—and by expressive factors—whether his vote was consistent with his norm. The parameter ε captures the weight of expressive relative to instrumental factors.

Voters cast their ballots simultaneously and the outcome of the vote is decided by majority rule. When determining equilibrium voting behavior, we restrict attention to symmetric equilibria. In that case, equilibrium is characterized by the voting behavior of each kind of voter, namely, voters with signals and types $(s,t) \in \{a,b\} \times \{A,B\}$. Absent expressive preferences (i.e., $\varepsilon = 0$), the model is quite standard and easy to analyze: all voters vote according to their signals in equilibrium and, for large n, the probability that the correct outcome is selected converges to one.[5]

We may divide voters into two classes depending on the realizations of s and t. When s and t coincide—that is, $s = a$ and $t = A$; or $s = b$ and $t = B$—we say that a voter is *unconflicted.* When s and t differ, we say that a voter is *conflicted.* After some simplification, it may be readily shown that the probability that a voter is conflicted is $\frac{1}{2}(1-q)$. Notice that when $q = 1$, type and signal are perfectly correlated and, as a consequence, there are no conflicted voters. As q falls, the probability that a voter is conflicted increases and reaches

[5]Because both states are equally likely ex ante, the usual worries about strategic voting highlighted by Austen-Smith and Banks (1996) are absent in this case.

a maximum of 50% at $q = 0$. Thus, in expectation, conflicted voters are always a minority in the voting population.

We now turn to voting strategy. We first show that voting for unconflicted voters is straightforward—they simply cast a vote consistent with both their signal and their type. In the proof of the following lemma—and in the remainder of the paper—γ_α denotes the equilibrium probability that a random voter casts a vote for A in state α. Likewise, γ_β denotes the probability that a random voter casts a vote for A in state β.

Lemma 1 *In all symmetric equilibria, unconflicted voters vote according to their type and signal.*

The lemma says that it never pays for unconflicted voters to forego their expressive payoffs *and* vote "strategically" (i.e., against their signal). The informational symmetry of the model nullifies the motives for strategic voting in the absence of expressive preferences and the additional cost in terms of expressive considerations merely reinforces this effect.

The voting behavior of conflicted voters is considerably more complex and interesting. Before proceeding with an equilibrium characterization, it is useful to define strategies more formally. Let σ_s denote the probability that a conflicted voter with signal s votes for A. From Lemma 1 it follows that

$$\gamma_\alpha = qr + r(1-q)\rho + r(1-q)(1-\rho)\sigma_a + (1-r)(1-q)\rho\sigma_b \tag{1}$$

$$\gamma_\beta = q(1-r) + (1-r)(1-q)\rho + (1-r)(1-q)(1-\rho)\sigma_a + r(1-q)\rho\sigma_b \tag{2}$$

Note that $\gamma_\beta < \gamma_\alpha$ for all $\{\sigma_a, \sigma_b\} \in [0,1]^2$. That is, A receives a greater (expected) share of the vote when it is the superior option than when it is the inferior option. The same is true for B. While $\{\sigma_a, \sigma_b\}$ describe a generic mixed strategy, two polar cases are of interest. When $\sigma_a = 1$ and $\sigma_b = 0$, we say that a voter votes instrumentally—that is, purely according to his signal. Similarly, when $\sigma_a = 0$ and $\sigma_b = 1$, we say that a voter votes expressively—that is, purely according to his type. Let $z_\theta = \gamma_\theta(1-\gamma_\theta)$, $\theta \in \{\alpha, \beta\}$. For a conflicted voter with signal s, the difference in expected payoffs between voting instrumentally and voting expressively takes on the same sign as V_s, where

$$\begin{aligned} V_a &\equiv \binom{n}{\frac{n}{2}} \left(r \left(z_\alpha \right)^{\frac{n}{2}} - (1-r) \left(z_\beta \right)^{\frac{n}{2}} \right) - \frac{\varepsilon}{1-\varepsilon}, \text{ and} \\ V_b &\equiv \binom{n}{\frac{n}{2}} \left(r \left(z_\beta \right)^{\frac{n}{2}} - (1-r) \left(z_\alpha \right)^{\frac{n}{2}} \right) - \frac{\varepsilon}{1-\varepsilon} \end{aligned}$$

Intuitively, instrumental payoff differences arise only when the vote is tied. They reflect the balance between tilting the vote toward the correct outcome given the signal, versus tilting it toward the incorrect outcome. Expressive payoff differences, on the other hand, always arise. Here, the term $\frac{\varepsilon}{1-\varepsilon}$ represents the (normalized) cost of voting against one's type.

3 Equilibrium Voting in Pure Strategies

Having characterized the equilibrium voting behavior of unconflicted voters, we now turn to the behavior of conflicted voters. As we show below, the behavior of conflicted voters typically varies with the size of the voting body. Intuitively, as the size of the voting body grows, instrumental considerations—which hinge on the probability of being pivotal—become less important and voting becomes more expressive.

While $n+1$ denotes the discrete size of the voting body, it is sometimes convenient to use a continuous analog of n, which we denote by m. We also adapt the usual floor/ceiling notation for the integer part of m to reflect the restriction that n be an even number. Specifically, let $\lfloor m \rfloor$ denote the largest even integer less than or equal to m, and let $\lceil m \rceil$ denote the smallest even integer greater than or equal to m. Finally, we use the Gamma function to extend factorials to non-integer values. Recall that, for integer values, $n! = \Gamma(n+1)$ and, hence, $\binom{n}{\frac{n}{2}} = \frac{\Gamma(n+1)}{\Gamma^2\left(\frac{n}{2}+1\right)}$. The expression $\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}$ represents the continuous analog. This continuous analog makes the function V_s—and similar expressions below—well-defined for all non-negative real-valued m.

We now offer a useful technical lemma which shows that, for fixed values of z_α and z_β, V_s is monotone in m. Formally,

Lemma 2 *Fix z_α and z_β such that $0 < z_\alpha \leq z_\beta \leq \frac{1}{4}$. Then*

$$\Phi(m) \equiv \frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left\{r\left(z_\beta\right)^{\frac{m}{2}} - (1-r)\left(z_\alpha\right)^{\frac{m}{2}}\right\}$$

is strictly decreasing in m. Moreover, $\lim_{m\to\infty}\Phi(m) \downarrow 0$.

Instrumental Equilibrium

From an information aggregation perspective, it would be ideal if voters simply voted in line with their signals. As we have shown above, this is not a problem for unconflicted voters. For conflicted voters, whether to vote instrumentally turns on whether the gains from improving the probability of breaking a tie in the right direction outweigh the losses from voting against one's expressive preferences.

Let z_α^I denote z_α under instrumental voting and note that $z_\alpha^I = z_\alpha|_{\sigma_a=1,\sigma_b=0} = r(1-r)$. Lemma 2 implies that the benefits from instrumental voting are strictly decreasing in m. Thus, finding the largest size voting body for which instrumental voting is an equilibrium simply amounts to determining the value of m such that $V_a|_{\sigma_a=1,\sigma_b=0} = V_b|_{\sigma_a=1,\sigma_b=0} = 0$ or, equivalently,

$$\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}(2r-1)\left(r(1-r)\right)^{\frac{m}{2}} = \frac{\varepsilon}{1-\varepsilon} \tag{3}$$

Lemma 2 also implies that, for all $m > 0$,

$$\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}(2r-1)\left(r(1-r)\right)^{\frac{m}{2}} < 2r-1$$

Hence, a necessary condition for instrumental voting to be an equilibrium for *some* size of the voting body is that $\frac{\varepsilon}{1-\varepsilon} < 2r-1$ or, equivalently, $\varepsilon < \frac{1}{r}\left(r-\frac{1}{2}\right)$. If $\varepsilon \geq \frac{1}{r}\left(r-\frac{1}{2}\right)$, voting expressively is the unique equilibrium, regardless of the size of the voting body. The remainder of the analysis excludes this rather uninteresting case. Formally,

Assumption 1: $\varepsilon < \frac{1}{r}\left(r-\frac{1}{2}\right)$.

Assumption 1 together with Lemma 2 guarantee that equation (3) has a unique solution in m, which we denote by $\overline{m}_I$. Hence, we have shown that

Proposition 1 *Instrumental voting is an equilibrium iff* $n \leq \overline{m}_I$.

Proposition 1 implies that, for large voting bodies, instrumental voting is not an equilibrium. Since the probability of being pivotal declines as the number of voters increase, the *effective* weight of instrumental payoffs, which depends on the chance of a tied election, declines relative to the effective weight of expressive payoffs. Once voters are sufficiently unlikely to swing the vote, they are better off voting according to their type and locking in the ε expressive utility, rather than voting according to their signal and foregoing this sure gain for a lottery with only a small chance of success.

Inspection of equation (3) reveals that $\overline{m}_I$ does not depend on q and ρ. That is, the size of the voting body for which instrumental voting is an equilibrium is independent of the prevalence of conflict between instrumental and expressive motives and the level of ex ante bias in expressive motives. The reason is that, under instrumental voting, the payoff from deviating and voting expressively is independent of q and ρ and, hence, so is the upper bound for instrumental voting, $\overline{m}_I$. Also, note that $\overline{m}_I$ varies non-monotonically with the quality of voters' information. When voters are poorly informed (i.e., $r < \frac{1}{2(1-\varepsilon)}$), instrumental voting is never an equilibrium. However, as voters become perfectly informed (i.e., $r \to 1$), $\overline{m}_I$ also goes to zero. There are two different forces at work here. When r is low, a voter is relatively likely to be pivotal but unlikely to push the outcome in the right direction. Hence, expected instrumental payoffs are low. When r is high, a voter is very likely to push the outcome in the right direction conditional on being pivotal, but very unlikely to be pivotal. Again, this leads to low expected instrumental payoffs. Thus, the size of the voting body for which instrumental voting is an equilibrium is largest when voters are moderately well-informed.

The fact that instrumental voting is not an equilibrium for voting bodies with more than $\lfloor \overline{m}_I \rfloor$ members might seem inconsequential provided that the weight on expressive payoffs is small. Indeed, inspection of equation (3) reveals that $\overline{m}_I$ becomes infinitely large as ε goes to zero. However, a key question is *how fast* the value of $\overline{m}_I$ grows as ε shrinks. While $\overline{m}_I$ does not have a closed-form solution, Stirling's approximation offers a way to examine the relationship between $\overline{m}_I$ and ε.

Remark 1 *For small ε,*

$$\overline{m}_I \approx \frac{1}{-\ln\left(4r\left(1-r\right)\right)} W\left(\frac{-\ln\left(4r\left(1-r\right)\right)}{\frac{\pi}{2}\left(\frac{1}{r}\frac{\varepsilon}{1-\varepsilon}\right)^2}\right) \tag{4}$$

where $W(\cdot)$ is the Lambert W function.[6]

Consider the sequence $\varepsilon_k = \frac{1}{k}$. Substituting this expression into equation (4) yields the sequence $\overline{m}_{I,k} \approx \xi \cdot W\left(\left(k-1\right)^2\right)$, where ξ is a scaling factor independent of k. Now recall that $\lim_{k\to\infty} \frac{\ln k}{W(k)} = 1$. Hence, we can conclude that as ε_k falls, $\overline{m}_{I,k}$ grows only at rate $2\ln k$. In other words, while $\overline{m}_{I,k}$ increases, it does so only extremely slowly.

Example 1 *Suppose that $r = \frac{3}{5}$ and $\varepsilon = 1/50$. Instrumental voting is an equilibrium for voting bodies of up to 23 voters. If $\varepsilon = 1/1000$, then $\lfloor \overline{m}_I \rfloor + 1$ increases to 129.*

Expressive equilibrium Let us now turn to the polar opposite case—expressive voting. Expressive voting is an equilibrium if and only if $\sigma_a = 0$ and $\sigma_b = 1$ is optimal for conflicted voters. This corresponds to $V_a|_{\sigma_a=0,\sigma_b=1} \leq 0$ and $V_b|_{\sigma_a=0,\sigma_b=1} \leq 0$. Let $z_\alpha^E \equiv z_\alpha|_{\sigma_a=0,\sigma_b=1}$, and let z_β^E be likewise defined. It may be readily verified that $z_\alpha^E < z_\beta^E$. Therefore,

$$\begin{aligned} V_b|_{\sigma_a=0,\sigma_b=1} &= \frac{\Gamma\left(m+1\right)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left\{r\left(z_\beta^E\right)^{\frac{m}{2}} - \left(1-r\right)\left(z_\alpha^E\right)^{\frac{m}{2}}\right\} - \frac{\varepsilon}{1-\varepsilon} \\ &> \frac{\Gamma\left(m+1\right)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left\{r\left(z_\alpha^E\right)^{\frac{m}{2}} - \left(1-r\right)\left(z_\beta^E\right)^{\frac{m}{2}}\right\} - \frac{\varepsilon}{1-\varepsilon} = V_a|_{\sigma_a=0,\sigma_b=1} \end{aligned}$$

Thus, we need only check the incentive condition for expressive voting for conflicted voters with b signals.

[6] Recall that the Lambert W function is the inverse of $f(W) = W\exp(W)$.

Because $z_\alpha^E < z_\beta^E$, Lemma 2 implies that the relative benefits from expressive voting are increasing in m. Hence, finding the smallest size voting body such that expressive voting is an equilibrium amounts to determining the value of m where $V_b|_{\sigma_a=0,\sigma_b=1} = 0$ or, equivalently,

$$\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left\{r\left(z_\beta^E\right)^{\frac{m}{2}} - (1-r)\left(z_\beta^E\right)^{\frac{m}{2}}\right\} = \frac{\varepsilon}{1-\varepsilon} \tag{5}$$

Together, Assumption 1 and Lemma 2 guarantee that equation (5) has a unique solution, which we denote by $\underline{m}_E$.[7] Hence,

Proposition 2 *Expressive voting is an equilibrium iff* $n \geq \underline{m}_E$.

One might have thought that $\underline{m}_E = \overline{m}_I$; that is, once instrumental voting ceases to be an equilibrium, expressive voting becomes an equilibrium. Notice, however, that this is generically not the case. This is most easily seen for $q = 0$. In that case, equation (5) reduces to

$$\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}(2r-1)(\rho(1-\rho))^{\frac{m}{2}} = \frac{\varepsilon}{1-\varepsilon}$$

Lemma 2 implies that $\underline{m}_E < \overline{m}_I$ if and only if $\rho > r$. Hence, instrumental and expressive equilibria may overlap, or there may be a gap between the two. The gap between $\overline{m}_I$ and $\underline{m}_E$ can be quite large indeed. To see this, let us return to the example above, filling in the remaining parameters of the model.

Example 2 *Let* $r = 3/5$, $\rho = 7/10$, $q = 7/10$, *and* $\varepsilon = 1/50$. *Instrumental voting is an equilibrium for* $n + 1 \leq 23$, *while expressive voting is an equilibrium for* $n + 1 \geq 459$.

This leaves open the question of what happens in between instrumental and expressive voting. The next section fills in this gap by considering mixed strategies.

[7] While $\underline{m}_E$ does not admit a closed-form solution, a good approximation is $\underline{m}_E \approx \frac{1}{-\ln\left(4z_\beta^E\right)} W\left(\frac{-\ln\left(4z_\beta^E\right)}{\frac{\pi}{2}\left(\frac{1}{r}\frac{\varepsilon}{1-\varepsilon}\right)^2}\right)$

4 Full Equilibrium Characterization

In this section, we allow for mixed strategies and characterize all symmetric equilibria. The following lemma narrows down the kind of voting behavior that can arise in equilibrium.

Proposition 3 *The following and only the following kinds of equilibria can arise:*

1) Instrumental, 2) completely mixed, 3) partially mixed, 4) expressive.

In a completely mixed equilibrium, conflicted voters strictly mix between instrumental and expressive voting. In a partially mixed equilibrium, conflicted voters with a signals vote expressively, while conflicted voters with b signals mix. The asymmetry stems from the fact that, since $\rho \geq 1/2$, voters are (weakly) biased toward voting for A.

To provide a full equilibrium characterization, it is convenient to distinguish between high and low q, where the threshold between the two, q^*, is defined in equation (10) in the Appendix. High q corresponds to a low prevalence of conflict between types and signals, such that instrumental and expressive motives tend to coincide. Low q corresponds to a high prevalence of conflict between types and signals. In that case, instrumental and expressive motives are more likely to be at odds with each other.

Low Conflict (High q) We now show that when conflict is low (i.e., $q > q^*$) , the intervals for which the various classes of equilibria exist partition the set of even integers. In the next proposition, the upper bound for completely mixed voting, $\overline{m}_{CM}$, is formally defined in equation (8) in the Appendix.

Proposition 4 *Under low conflict, there exists a unique equilibrium for each n. Equilibrium is: 1) Instrumental for $n \leq \overline{m}_I$, 2) completely mixed for $\overline{m}_I < n < \overline{m}_{CM}$, 3) partially mixed for $\overline{m}_{CM} \leq n < \underline{m}_E$, 4) expressive for $n \geq \underline{m}_E$.*

Proposition 4 establishes that, as n increases, equilibrium moves smoothly from instrumental to expressive voting. When a voting body is small, instrumental voting is the unique equilibrium. As the voting body grows larger, equilibrium voting becomes completely mixed. As it grows larger yet, we move to partially mixed voting. That is, voters with a signals vote expressively while voters with b signals continue to mix. Finally, in sufficiently large voting bodies, expressive voting is the unique equilibrium.

We say that voting becomes more expressive if σ_a decreases and σ_b increases and show that

Proposition 5 *Under low conflict, equilibrium voting becomes more expressive as n increases.*

Finally, let us return to Example 2. Because $q = \frac{7}{10} > q^*$, we are in the low conflict case and the analysis above applies. Recall that, for the parameter values in the example, instrumental voting is an equilibrium for 23 voters or less, while expressive voting is an equilibrium for 459 voters or more. Completely mixed voting is an equilibrium for voting body sizes of 25 and 27, while partially mixed voting is an equilibrium for sizes between 29 and 457.

High Conflict (Low q) We now turn to the case where conflict between types and signals is high (i.e., $q < q^*$). As we shall see, this makes equilibrium behavior more complex. While the classes of equilibria are the same as under low conflict, under high conflict, the ranges for which these classes exist may overlap. Indeed, instrumental and expressive equilibria may coexist for the same value of n. Moreover, equilibrium may no longer be unique within a class: for generic parameter values, two different partially mixed equilibria coexist, which we call "high" partially mixed and "low" partially mixed. While a conflicted voter with an a signal votes expressively in both partially mixed equilibria, the equilibria differ in the probability that a conflicted voter with a b signal votes expressively. In a high partially mixed equilibrium, this probability is relatively large, while it is relatively small in a low partially mixed equilibrium.

In the next proposition, the upper bound for partially mixed voting, $\overline{m}_{PM}$, is formally defined in equation (13) in the Appendix.

Proposition 6 *Under high conflict (i.e., $q < q^*$), equilibria are: 1) Instrumental iff $n \leq \overline{m}_I$, 2) completely mixed iff $\overline{m}_I < n < \overline{m}_{CM}$, 3) low partially mixed iff $\overline{m}_{CM} \leq n < \overline{m}_{PM}$, 4) high partially mixed iff $\underline{m}_E \leq n < \overline{m}_{PM}$, 5) expressive iff $n \geq \underline{m}_E$.*

Moreover, within each (sub-)class, equilibrium is unique.

While, typically, expressiveness increases with the size of the voting body, the sequence of high partially mixed equilibria has the somewhat counter-intuitive property that expressiveness *decreases* with n. Moreover, for some parameter values and voting body sizes, instrumental and expressive equilibria coexist. To see this, consider the following amendment of Example 2, where we have reduced q from 7/10 to 1/10.

Example 3 *Let $r = 3/5$, $q = 1/10$, $\rho = 7/10$, and $\varepsilon = 1/50$. Instrumental voting is an equilibrium for $n + 1 \leq 23$, while expressive voting is an equilibrium for $n + 1 \geq 19$. There is a completely mixed equilibrium for $25 \leq n + 1 \leq 43$, a low partially mixed equilibrium for $45 \leq n + 1 \leq 549$, and a high partially mixed equilibrium for $21 \leq n + 1 \leq 549$.*

5 The Optimal Size of Voting Bodies

What is the optimal size of voting bodies? Having characterized equilibrium behavior for bodies of all sizes, we are now in a position to formally address this question. Our preferred metric is selection accuracy, S. That is, the probability that a voting body chooses the (instrumentally) correct outcome given the state.

To the extent that instrumental motives reflect "reason" and expressive motives "passion," Madison's view that large voting bodies lead to the ascendancy of passion over reason proved to be correct: with the exception of the high partially mixed equilibrium, equilibrium voting becomes more expressive, when the size of the voting body increases. Thus, the key trade-off for accuracy is between the informational gains from adding an additional voter versus the informational losses from more expressive voting. Implicit in Madison's arguments against direct democracy is the idea that, at some point, the latter effect dominates the former. As we shall see, whether this really happens depends on the prevalence of conflict between expressive and instrumental motives.

Fix an equilibrium $(\gamma_\alpha, \gamma_\beta)$ for a voting body of size $n+1$. In state α, the equilibrium probability that an individual voter casts a vote for the correct outcome, A, is γ_α. Therefore, the voting body selects the correct outcome with probability

$$S(n+1|\alpha) = \sum_{k=\frac{n}{2}+1} \binom{n+1}{k} \gamma_\alpha^k (1-\gamma_\alpha)^{n+1-k}$$

In state β, the equilibrium probability that an individual voter casts a vote for the correct outcome, B, is $1-\gamma_\beta$. Thus, the voting body selects the correct outcome with probability

$$S(n+1|\beta) = \sum_{k=\frac{n}{2}+1} \binom{n+1}{k} (1-\gamma_\beta)^k \gamma_\beta^{n+1-k}$$

Since the two states are equally likely ex ante, $S(n+1) = \frac{1}{2}(S(n+1|\alpha) + S(n+1|\beta))$.

It is sometimes convenient to extend S to non-integer values, m. Since the cdf of a binomial distribution may be expressed in terms of Beta functions (see, e.g., Press *et al.*, 1992), we have*:*

$$S(m+1) = \frac{1}{2}\left(\frac{B\left(\gamma_\alpha, \frac{m}{2}+1, \frac{m}{2}+1\right)}{B\left(\frac{m}{2}+1, \frac{m}{2}+1\right)} + \frac{B\left(1-\gamma_\beta, \frac{m}{2}+1, \frac{m}{2}+1\right)}{B\left(\frac{m}{2}+1, \frac{m}{2}+1\right)}\right)$$

Here, $B(x, y)$ denotes the Beta function with parameters x and y, while $B(\gamma, x, y)$ denotes the incomplete Beta function.

Low Conflict (High q) Suppose that the prevalence of conflict between instrumental and expressive motives is low (i.e., $q > q^*$). In that case, there is a unique equilibrium for each size voting body and, hence, $S(n+1)$ is uniquely determined. Once the voting body becomes sufficiently large, voting is purely expressive. Beyond this point, there is no more trade-off between information and expressiveness. As only the informational force persists, it might seem that information should always aggregate in the limit.

To see that this is not the case, consider the informational value of a marginal voter when voting is purely expressive. In state α, the chance that a voter casts a vote for the correct outcome is $\gamma_\alpha^E = qr + (1-q)\rho$. Since $r > \frac{1}{2}$ and $\rho \geq \frac{1}{2}$, we have $\gamma_\alpha^E > \frac{1}{2}$. This means that the marginal voter always improves accuracy in state α. In state β, the chance that a voter casts a vote for the correct outcome is $1 - \gamma_\beta^E = 1 - \rho + q(r + \rho - 1)$. The marginal voter improves accuracy if and only if $1 - \gamma_\beta^E > \frac{1}{2}$. Hence, the threshold value of q such that the informational contribution is positive in state β is $q > \frac{1}{2}\frac{2\rho-1}{\rho-(1-r)} \equiv q_1$. Therefore, in the limit, the probability of selecting the correct outcome in state α always goes to one. The probability in state β goes to one if and only if $q > q_1$, where, as proved in Lemma 7 in the Appendix, $q^1 > q^*$. We have established that

Proposition 7 *In large voting bodies, information fully aggregates if and only if conflict is very low, i.e., $q > q_1$, where $q_1 > q^*$.*

What happens when conflict is low, but not *very* low (i.e., $q^* < q < q_1$)? Because $\gamma_\alpha^E > \frac{1}{2}$ and $1 - \gamma_\beta^E < \frac{1}{2}$, each incremental voter increases the chance of selecting the correct outcome in state α, but decreases it in state β. Since $\gamma_\alpha^E > \gamma_\beta^E > \frac{1}{2}$, γ_α^E is farther from $\frac{1}{2}$ than is $1 - \gamma_\beta^E$. This means that the incremental voter is more likely to break a tie correctly in state α than he is to break a tie *in*correctly in state β. On the other hand, it also means that the probability of a tie is greater in state β than in state α. When n is small, tie probabilities are relatively similar across states and, hence, adding a voter is beneficial. When n is large, ties are vastly more likely in state β and, thus, the marginal voter has a negative effect on accuracy. In the limit, the correct outcome is chosen with probability one in state α, but is *never* chosen in state β. As a result, accuracy falls to 50%. Formally,

Proposition 8 *Suppose conflict is not very low (i.e., $q^* < q < q_1$). Then, for n sufficiently large, the incremental voter has negative informational value. That is, $S(n+1)$ is eventually*

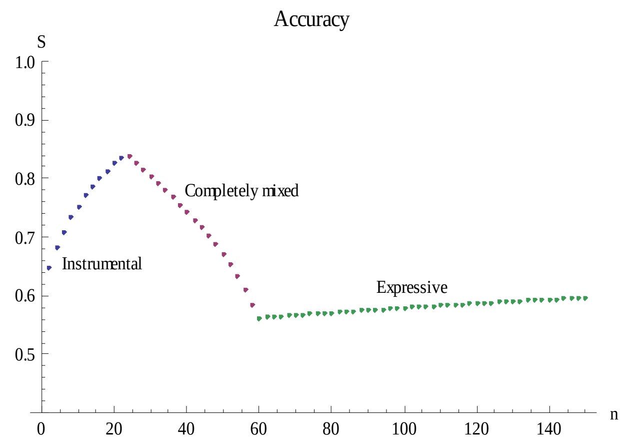

Figure 1: Accuracy under low conflict (i.e., high q).

decreasing in n. Furthermore, in the limit, large voting bodies are no better than a coin flip at selecting the correct outcome.

Proposition 8 may be seen as a formalization of Madison's intuition that "passion" leads voting bodies to "counteract their own views by every addition to their representatives" (Federalist No. 58). Unless conflict is very low, eventually, each additional voter reduces accuracy, despite the fact that voters' preferences are instrumentally aligned.

It might seem that when conflict *is* very low, the best strategy is to always make the voting body as large as possible. Indeed, when n is sufficiently large, incremental voters have positive informational value and, hence, locally, their addition is unambiguously helpful. For smaller values of n, however, the trade-off between information and expressiveness is still present, and the contribution of incremental voters may very well be negative. This holds even when there is no ex-ante asymmetry in norms (i.e., $\rho = 1/2$). Specifically,

Proposition 9 *For all q, accuracy is strictly decreasing in the region of the completely mixed equilibrium. Formally, for $\overline{m}_I < n < \overline{m}_{CM}$, $S(n+1)$ is strictly decreasing in n.*

To illustrate the potential importance of this effect, we offer an example where the "valley" of larger voting bodies producing lower accuracy is considerable. Suppose that we amend Example 3 to remove any asymmetry in ex ante norms (i.e., $\rho = 1/2$). Since $q = 1/10 > 0 = q_1$, equilibrium is unique for every n and accuracy converges to 1 in the limit. However, as Figure 1 illustrates, increasing the number of voters is not the same as increasing accuracy. While accuracy increases along the instrumental equilibrium sequence (up to $n+1 = 23$), it falls along the completely mixed equilibrium sequence (between $n+1 = 25$ and 61). Beyond this point, accuracy once again increases, but it only reaches its previous high water mark at $n + 1 = 2{,}429$. In the region of the completely mixed equilibrium, an increase in the size of the voting body leads to informational losses from more expressive voting that outpace the informational gains from having more voters. From $n + 1 = 61$ onwards, voting is purely expressive and the informativeness of votes no longer degenerates as n increases.[8] Since $q > q_1$, additional votes improve equilibrium accuracy, albeit slowly. The point is that, even when conflict is very low, expanding the voting body is not necessarily conducive to obtaining better policies.

High Conflict (Low q) When conflict is high (i.e., $q < q^*$), equilibrium multiplicity complicates the determination of the optimal size of voting bodies, as accuracy depends on which equilibrium is selected. Amending our notation, let $S_\eta(n+1)$ denote the selection accuracy of an equilibrium of type $\eta \in \{I, CM, LPM, HMP, E\}$. Here, I, CM, LPM, HMP, and E denote instrumental, completely mixed, low partially mixed, high partially mixed, and expressive equilibrium, respectively. The next proposition shows that, if different types of equilibria coexist for a voting body of a given size, then they can be ordered in terms of accuracy.

Proposition 10 *If multiple equilibria coexist for given n, then their ranking in terms selection accuracy is:*

$$S \in \{S_I, S_{CM}, S_{LPM}\} > S_{HPM} > S_E$$

[8]When $\rho = \frac{1}{2}$, the partially mixed equilibrium region disappears as a consequence of the symmetry of the model.

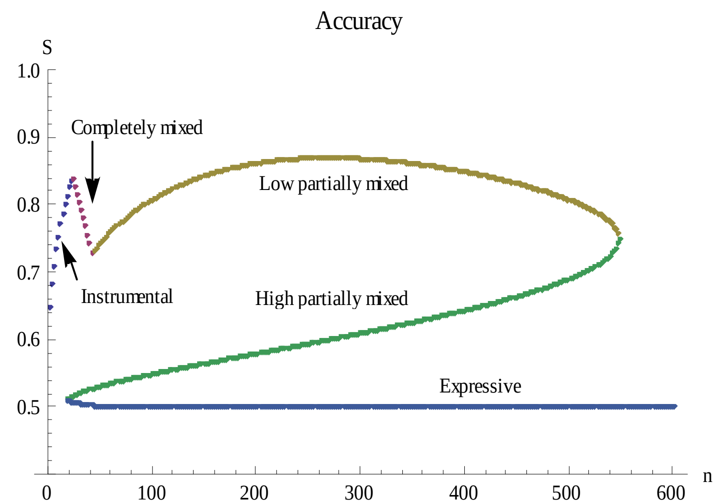

Figure 2: Accuracy under high conflict (i.e., low q).

Proposition 10 is intuitive: the accuracy ranking corresponds to the expressiveness of equilibria. Thus, an expressive equilibrium is least accurate, while an instrumental equilibrium—provided one exists for the same size voting body—is most accurate. Other equilibria are similarly ordered.

It can be easily verified that Proposition 8 carries over to high-conflict environments (i.e., $q < q^*$). Hence, in large voting bodies, the incremental voter has negative informational value and, in the limit, voting bodies are no better than a coin flip at selecting the correct outcome. For small voting bodies, however, increasing size can increase accuracy. When instrumental voting is an equilibrium, adding more voters is obviously helpful. But even when voting is expressive, initially, adding voters may improve accuracy. This happens as long as the likelihood of a tie remains comparable between the two states.

Accuracy properties under high conflict are illustrated in Figure 2. The figure depicts the selection accuracy of the equilibria in Example 3 as a function of n. As the figure illustrates, accuracy is increasing in n under instrumental voting and decreasing under completely mixed voting. Accuracy is hump-shaped under low partially mixed voting, increasing under high partially mixed voting and, eventually, decreasing under expressive voting.

Figure 2 also illustrates that, under high conflict, equilibrium accuracy can drop *discontinuously* in n. In other words, accuracy does not degrade "gracefully" as the voting body grows but, at some point, falls off a cliff. Let us denote the sequence of most informative equilibria by $C(m)$, where we treat m as continuous. From Proposition 10 we know that this sequence (function) is uniquely defined even in the presence of multiple equilibria. Next, notice that at $m = \overline{m}_{PM}$, $C(m)$ moves from low partially mixed to expressive voting. Moreover, from Proposition 10 we know that, for fixed m, $S_E(m+1) < S_{LMP}(m+1)$. Thus, we have shown:

Proposition 11 *Suppose voters coordinate on the most accurate equilibrium. Then, under high conflict, accuracy falls discontinuously at* $\overline{m}_{PM}$.

While accuracy falls discontinuously at $\overline{m}_{PM}$ when equilibrium selection is optimistic, notice that, under high conflict, accuracy must fall discontinuously at some point, regardless of the equilibrium selection rule.

Summary Unless conflict is very low, large voting bodies are highly undesirable as they do no better than a coin flip at selecting the correct outcome. While smaller voting bodies do better, they can experience a sudden discrete drop in accuracy when the size of the voting body is expanded, or when the prevalence of conflict between instrumental and expressive preferences rises. This is true even if we assume that voters are always able to coordinate on the "best" equilibrium.

When conflict is very low, information fully aggregates in the limit. This does not mean, however, that enlarging the voting body is necessarily a good idea. The reason is that accuracy is non-monotone in size. Therefore, unless the number of additional voters is sufficiently large, enlarging the voting body may reduce accuracy.

Minimally Expressive Preferences When expressive preferences are absent (i.e., $\varepsilon = 0$), our model is a standard Condorcet jury model in which information fully aggregates in the limit. On that basis, one might conjecture that, for small ε, large voting bodies produce outcomes that are close to optimal. Our final result shows that this is not the case. Even when the weight on expressive payoffs becomes arbitrarily small, equilibrium accuracy under mixed motives may not approach accuracy under purely instrumental motives as the voting body grows. To see this, fix a sequence $\{\varepsilon_k\} \to 0$. For each element of this sequence, let S_{ε_k} denote asymptotic selection accuracy as $n \to \infty$. For n large, expressive voting is the unique equilibrium. Hence, $S_{\varepsilon_k} = \lim_{n\to\infty} S_E(n+1)$. Finally, let S^* denote the asymptotic selection accuracy as $n \to \infty$ for $\varepsilon = 0$. It is easy to show that $S^* = 1$. Using Proposition 7 it then follows that

Proposition 12 *Unless conflict is very low, the accuracy of large voting bodies as $\varepsilon \to 0$ does not converge to accuracy of large voting bodies when $\varepsilon = 0$. Formally, if $q < q_1$ then, for every sequence $\{\varepsilon_k\} \to 0$, $\{S_{\varepsilon_k}\} \to \frac{1}{2} < 1 = S^*$.*

The discontinuity arises from the fact that we consider asymptotic accuracy as $n \to \infty$ for fixed ε and only then let ε go to zero. If, instead, we reversed the order of limits, information would fully aggregate. However, since our concern is with the accuracy of large voting bodies for ever smaller values of ε, the former order of limits is the appropriate one.

Other Measures of Welfare By using accuracy, S, to determine the optimal size of voting bodies, we implicitly assumed that, from a societal point of view, only instrumental payoffs matter. Of course, this neglects the expressive payoffs of members of the voting body. It also neglects the possibility that, in fact, expressive payoffs may be the most relevant.

If one chose to include expressive payoffs among the benefits, our conclusions would remain unaltered. To see this, note that in instrumental and expressive equilibria, a voter's expressive payoffs are unaffected by the size of the voting body. Hence, accuracy is the sole determinant of welfare. In completely mixed and partially mixed equilibria, voters who mix are indifferent between voting expressively and voting instrumentally. Thus, for purposes of payoff comparison, we may assume they vote expressively. Receiving full expressive payoffs, accuracy is then again the sole determinant of these voters' welfare as the size of the voting body changes. The same is true for voters in completely and partially mixed equilibria who do not mix, because their expressive payoffs are again unaffected by the size of the voting body.

Sometimes, only expressive payoffs should be counted from a societal point of view. Consider, for instance, situations where instrumental payoffs correspond to rent-seeking benefits while expressive payoffs reflect ethical concerns about appropriate policy. In that case, it may well be that social welfare is maximized by aggregating expressive preferences and ignoring instrumental concerns. In such situations, our results operate in reverse: when conflict is high, the problem is with voting bodies that are too small rather than too large. Money will override conscience in small voting bodies, while the "better angels of our nature" will dictate voting in large bodies.

Expressive Preferences and the Probability of Being Pivotal Once the probability of casting a decisive vote falls sufficiently, expressive motives completely crowd out instrumental motives. Hence, one might suspect that pivotality considerations play a subordinated role in our model more generally. This, however, is not the case.

Figure 3 illustrates the probability of being pivotal in Example 3 as n increases. While the pivot probability falls rapidly under instrumental voting, it is constant under completely mixed voting. The reason is that expressive payoffs do not depend on n. To keep conflicted voters mixing, instrumental payoffs then cannot depend on n either. Hence, the probability of being pivotal must remain constant when n increases.[9] Next, as conflicted voters with a signals stop mixing, initially, the pivot probability falls under partially mixed voting, but then remains stubbornly high, because conflicted voters with b signals continue to mix.[10]

[9] The probability of being pivotal in the completely mixed equilibrium is $\frac{1}{2r-1}\frac{\varepsilon}{1-\varepsilon}$.

[10] The pivotal probability in the low and high partially mixed equilibria converges to $\frac{1}{2r}\frac{\varepsilon}{1-\varepsilon}$.

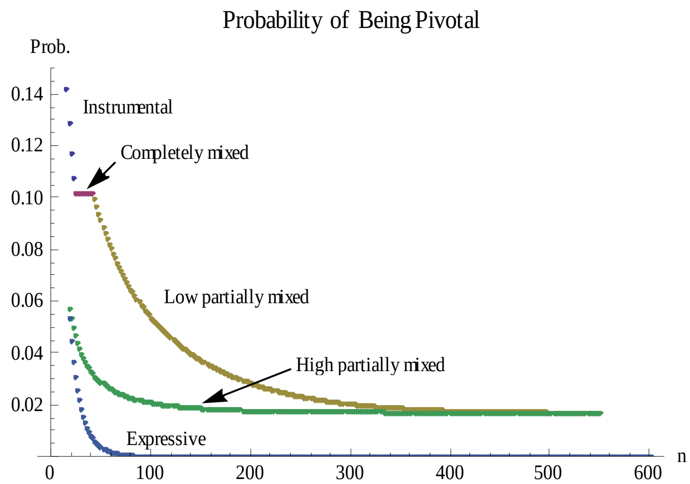

Figure 3: Pivot probabilities as a function of n.

In the example, the pivot probability under partially mixed voting stays above 1.7%, even when $n+1$ is as high as 549. As a comparison, under purely instrumental preferences (i.e., $\varepsilon = 0$), the pivot probability for $n+1 = 549$ is 2.8×10^5 times smaller. Beyond $\overline{m}_{PM}$ only expressive voting is an equilibrium, and the chance of being pivotal falls discontinuously to, essentially, zero.

The large difference in pivot probabilities between $\varepsilon > 0$ and $\varepsilon = 0$ does not depend on high rates of conflict. To see this, note that in Example 2 the pivot probability for $n_E + 1 = 459$ under partially mixed voting is again 1.7%, while the pivot probability under $\varepsilon = 0$ is 1.9×10^4 times smaller.

Preplay Communication It is well-known that preplay communication can have a large effect on equilibrium voting behavior and accuracy. Indeed, when voters have common interests and preferences are purely instrumental, a simple straw poll "solves" the voting problem regardless of the particular voting rule used (see, e.g., Coughlan, 2000). In that setting, the straw poll allows all voters to have the same information and, hence, the same preferences prior to the vote. In our setting, preplay communication also allows for coordination among voters. To see this most clearly, consider the polar case where $r = 1$—i.e., all voters are perfectly informed about the state. Despite the fact that the state is common knowledge, absent preplay communication, the vote may still produce the wrong outcome if q is sufficiently low. Introducing preplay communication avoids this possibility. For instance, the following is an equilibrium in the game with preplay communication: All voters truthfully exchange information about their expressive preferences and those whose expressive preferences coincide with the state vote accordingly. If the number of votes is insufficient to implement the efficient outcome, choose a minimum winning coalition that includes a minimal number of voters with opposing expressive preference and ask that these voters also vote for the efficient outcome. Since, in equilibrium, every voter is pivotal with probability one, instrumental motives dominate even in large elections, and the efficient outcome is selected.

It would seem that preplay communication offers a similar solution for r close to one. However, determining whether information exchange is incentive compatible becomes more problematic. When voters have mixed motives, an election is not a pure common interest game and, therefore, it is not clear that voters would wish to reveal their imperfect signals truthfully in a straw poll. In addition to this strategic complication, with mixed motives, preplay communication introduces a host of other difficulties. First, preplay communication need no longer represent pure cheap talk. Indeed, voters may derive direct (dis)utility from their votes in the straw poll, which would have to be modeled. Second, we argued that expressive preferences are malleable—it is possible for new information to change a voter's view about the appropriate norm. Hence, the outcome of the straw poll may affect voters' expressive preferences. Moreover, anticipating this effect, voters may want to strategically adjust their strategies in the straw poll. Finally, in our model, voters cast only a single vote and, therefore, consistency does not arise as an expressive consideration. However, once multiple votes are taken, expressive payoffs might well depend on each vote separately, as well as on the *combination* of votes cast. For example, a voter might experience losses from "flip-flopping" at successive stages. All of this considerably complicates the modeling and analysis of preplay communication in the presence of mixed motives. While preplay communication is of obvious of interest in determining the optimal size of voting bodies, a full analysis is beyond the scope of the present paper.

6 Conclusion

Since Condorcet, perhaps the main message from the "informational" voting literature is the remarkable ability of elections to aggregate information and produce the correct decision. Our analysis suggests that, perhaps, we have been overly optimistic in these conclusions. When we enrich the classical model by admitting the possibility that voters are motivated by expressive as well as instrumental motives, the results are more ambiguous and the conclusions less hopeful.

Expressive motives fundamentally change equilibrium behavior and subsequent conclusions about the quality of decisions made through voting. In a sense, purely instrumental models of voting represent a "singularity:" even if the weight put on expressive preferences is arbitrarily small, they completely determine voting behavior in large voting bodies.

Whether this is for good or for ill depends on how malleable—influenced by facts—expressive preferences are. As long as they are sufficiently malleable, expressive voting is of no real concern, as it still leads to the correct outcome in the limit. In contrast, when expressive preferences are rigid and relatively impervious to facts, expressive voting produces dismal results in large voting bodies. In the limit, information is driven out entirely and decisions are no better than chance. It is a situation James Madison would have recognized. Indeed, he observed that large voting bodies "counteract their own views by every addition to their representatives" (Federalist No. 58).

However, even when expressive preferences are malleable, there is always a region where informational losses from increased expressive voting dominate the informational gains from adding more voters. The reason is that, while the marginal voter does provide additional information, increased expressiveness drives out instrumentality over the entirety of the voting body. When expressive preferences are more rigid, this effect need not even be gradual: as the voting body expands in size, at some point, there will be a sudden downward jump in performance.

A practical implication of our results is that capping the size of voting bodies may be desirable even when logistical or information acquisition constraints are not binding. Thus, our model offers a rationale for the more or less constant size of many legislatures, despite significant population growth and advances in communication and information gathering technologies. Our results also highlight an important downside to increasing the transparency of the voting records of elected representatives. To the extent that transparency increases the need to pander to constituents, it increases expressiveness and, thereby, can have a deleterious effect on the performance of legislatures.

Of course, our model is merely suggestive in this regard. While the Condorcet setup offers a parsimonious framework for examining the effect of expressive motives on voting behavior and performance, it is by no means a complete description of real-world legislative settings. One obvious omission is the role of ideology on decision making. In our model, voters have common values—ex post all agree about the best choice. While there are some situations, such as national defense, where this may be a reasonable approximation, it is clearly inadequate in many other circumstances, such as when deciding on the appropriate role of government in regulating behavior. Similarly, in our model, all voters bear the same cost of errors. Yet, in many situations, voters may differ along this dimension as well. If one thinks about choices as reflecting conservative versus liberal policies in the service of some commonly agreed upon objective, then it may well be that some voters will find it more costly to be wrong when selecting the conservative policy, while others suffer more when an incorrect liberal policy is implemented. Finally, voters might also differ in the weight they place on expressive preferences. For instance, legislators with safe seats might place less weight on expressive considerations than those whose seats are more hotly contested. Clearly, a richer model would allow greater scope for ideology and other differences across voters. This, however, remains for the future.

A Proofs

Proof of Lemma 1

Consider an unconflicted voter with an a signal. Suppose, contrary to the statement of the lemma, that he prefers to vote for B rather than A. That is,

$$\binom{n}{\frac{n}{2}}(1-\varepsilon)\left(r(z_\alpha)^{\frac{n}{2}}-(1-r)(z_\beta)^{\frac{n}{2}}\right)+\varepsilon\leq 0 \tag{6}$$

where $z_\theta=\gamma_\theta(1-\gamma_\theta)$.

First, note that a necessary condition for this inequality to hold is that $z_\beta>z_\alpha$. Second, note that the inequality implies that a conflicted voter with an a signal would also strictly prefer to vote for B, i.e., $V_a<0$. Furthermore, an unconflicted voter with a b signal would strictly prefer to vote for B. To see this, note that the difference in that voter's payoff from voting for B rather than A is

$$\binom{n}{\frac{n}{2}}(1-\varepsilon)\left(r(z_\beta)^{\frac{n}{2}}-(1-r)(z_\alpha)^{\frac{n}{2}}\right)+\varepsilon$$

and this expression is strictly positive, since $z_\beta>z_\alpha$ and $r>\frac{1}{2}$. Finally, a conflicted voter with a b signal would strictly prefer to voter for B since $V_b>-V_a>0$.

Hence, we have shown that, if an unconflicted voter with an a signal weakly prefers to vote for B, then all voters strictly prefer to vote for B. In turn, this implies that $z_\alpha\geq z_\beta$; this, however, contradicts $z_\beta>z_\alpha$.

The proof that an unconflicted voter with a b signal strictly prefers to vote for B is analogous.

Proof of Lemma 2

Differentiating $\Phi(m)$ with respect to m, we obtain

$$\Phi'(m) = \frac{1}{2}\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left\{\begin{array}{c}(1-r)(z_\alpha)^{\frac{m}{2}}\left(2H\left[\frac{m}{2}\right]-2H[m]-\log[z_\alpha]\right)\\ -r(z_\beta)^{\frac{m}{2}}\left(2H\left[\frac{m}{2}\right]-2H[m]-\log[z_\beta]\right)\end{array}\right\}$$

where $H[x]$ is the xth harmonic number. Note that $\Phi'(m)$ takes the sign of the expression in curly brackets.

We claim that $2\left(H\left[\frac{m}{2}\right]-H[m]\right)-\log[z_\beta]>0$, for all $m\geq 2$. When $m=2$, we have

$$2(H[1]-H[2])-\log[z_\beta]>2(H[1]-H[2])-\log\left[\frac{1}{4}\right]=2\left(1-\frac{3}{2}\right)-\log\left[\frac{1}{4}\right]>0$$

Because $H[m]$ is concave in m, the inequality then also holds for all $m>2$.

This implies that $\Phi'(m)<0$ iff

$$\frac{1-r}{r}\frac{2\left(H[m]-H\left[\frac{m}{2}\right]\right)+\log[z_\alpha]}{2\left(H[m]-H\left[\frac{m}{2}\right]\right)+\log[z_\beta]}<\left(\frac{z_\beta}{z_\alpha}\right)^{\frac{m}{2}}$$

And this inequality indeed holds, because $r<\frac{1}{2}$ and $z_\alpha\leq z_\beta$.

To establish the second part of the lemma, use Stirling's approximation to obtain

$$\Phi(m)\approx\sqrt{2}\left(r\frac{\left(2\sqrt{z_\beta}\right)^m}{\sqrt{\pi m}}-(1-r)\frac{\left(2\sqrt{z_\alpha}\right)^m}{\sqrt{\pi m}}\right)$$

for large m. Now note that both terms converge to zero as $m\to\infty$, because $z_\alpha\leq z_\beta\leq\frac{1}{4}$. Hence, $\lim_{m\to\infty}\Phi(m)=0$.

Proof of Proposition 1

A necessary and sufficient condition for instrumental voting to be an equilibrium is that

$$\binom{n}{\frac{n}{2}}(2r-1)(r(1-r))^{\frac{n}{2}}\geq\frac{\varepsilon}{1-\varepsilon} \tag{7}$$

Note that lemma 2 with $z_\alpha=z_\beta=r(1-r)$ implies that the LHS is strictly decreasing in n. As a consequence, the inequality (7) holds iff $n\leq\overline{m}_I$, where $\overline{m}_I$ is the value of m that solves the continuous analogue of (7) with equality.

Proof of Proposition 2

Under expressive voting, $\sigma_a = 0$ and $\sigma_b = 1$. It may be readily verified that this implies that $z_\alpha < z_\beta$ and, therefore, $V_a < V_b$. Thus, we need only check the incentive condition to vote expressively for conflicted voters with b signals, i.e., $V_b \leq 0$. By construction, $V_b = 0$ at $\underline{m}_E$ while, by Lemma 2, V_b is strictly decreasing in n. Hence, the incentive constraint also holds for all $n \geq \underline{m}_E$.

Proof of Proposition 3

The fact that each of these kinds of equilibria can indeed arise is proved by example. (See, for instance, Example 3.) The proof that no other kinds of equilibria can arise proceeds as follows. First, from Lemma 1, we know that all unconflicted voters vote according to their signals. This implies that all equilibria are fully characterized by the mixing probabilities $(\sigma_a, \sigma_b) \in [0,1]^2$ of conflicted voters. To prove the proposition, we have to show that there neither exist equilibria with $\{\sigma_a = 1,\ \sigma_b \in (0,1)\}$, nor with $\{\sigma_a \in (0,1)\,,\ \sigma_b = 1\}$, nor with $\{\sigma_a \in (0,1)\,,\ \sigma_b = 0\}$. This is proved in Lemmas 3, 4, and 5 below.

Lemma 3 *There is no partially mixed equilibrium with $\sigma_a = 1$ and $\sigma_b \in (0,1)$.*

Proof. Suppose, by contradiction, that such an equilibrium does exist.

We first show that $\sigma_a = 1$ implies $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$. One may readily verify that for $\sigma_a = 1$, $\gamma_\alpha > \frac{1}{2}$. Furthermore, $\gamma_\beta > \frac{1}{2}$ iff $\sigma_b > \frac{r-\frac{1}{2}}{r(1-q)\rho}$. When $\sigma_b > \frac{r-\frac{1}{2}}{r(1-q)\rho}$, $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$ follows immediately from $\frac{1}{2} < \gamma_\beta < \gamma_\alpha$. When $\sigma_b \leq \frac{r-\frac{1}{2}}{r(1-q)\rho}$, $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$ is equivalent to showing that $\gamma_\alpha - (1-\gamma_\beta) > 0$. And after some algebra, $\gamma_\alpha - (1-\gamma_\beta) = (1-q)\,\rho\sigma_b > 0$.

Since $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$, we have $z_\alpha < z_\beta$ and, therefore, $V_b > V_a$. Because $\sigma_a = 1$, it must be that $V_a \geq 0$, which implies $V_b > V_a \geq 0$. Thus, conflicted voters with b signals strictly prefer to vote instrumentally, such that $\sigma_b = 0$. But his is a contradiction because, by assumption, $\sigma_b \in (0,1)$. ■

Lemma 4 *There is no partially mixed equilibrium with $\sigma_a \in (0,1)$ and $\sigma_b = 1$.*

Proof. Suppose, by contradiction, that such an equilibrium does exist. We first show that $\sigma_b = 1$ implies $z_\alpha < z_\beta$. The algebra establishing this is straightforward and analogous to that given in the proof of Lemma 3. Since $z_\alpha < z_\beta$, we have $V_b > V_a$. Because $\sigma_b = 1$, it must be that $V_b \leq 0$, which implies $V_a < V_b \leq 0$. Thus, conflicted voters with a signals strictly prefer to vote expressively, such that $\sigma_a = 0$. This is a contradiction because, by assumption, $\sigma_a \in (0, 1)$. ■

Lemma 5 *There is no partially mixed equilibrium with* $\sigma_a \in (0, 1)$ *and* $\sigma_b = 0$.

Proof. Suppose, by contradiction, that such an equilibrium does exist. We first show that $\sigma_b = 0$ implies $z_\alpha > z_\beta$. The algebra establishing this is straightforward and analogous to that given in the proof of Lemma 3. Since $z_\alpha > z_\beta$, we have $V_b < V_a$. Because $\sigma_b = 0$, it must be that $V_b \geq 0$, which implies $V_a > V_b \geq 0$. Thus, conflicted voters with a signals strictly prefer to vote instrumentally, such that $\sigma_a = 1$. This is a contradiction because, by assumption, $\sigma_a \in (0, 1)$. ■

This completes the proof of Proposition 3.

Proof of Proposition 4

The proof follows from a sequence of lemma's and propositions. We first consider completely mixed equilibria. The following lemma identifies properties that all such equilibria share. Denote the probability of being pivotal in state θ by $\Pr[piv|\theta]$. Then,

Lemma 6 *In any completely mixed equilibrium, 1)* $\Pr[piv|\alpha] = \Pr[piv|\beta] = \frac{1}{2r-1}\frac{\varepsilon}{1-\varepsilon}$, *2)* $\gamma_\alpha = 1 - \gamma_\beta > \frac{1}{2}$, *3)* $\sigma_a = 1 - \frac{\rho}{1-\rho}\sigma_b$.

Proof. The probability of being pivotal in state θ is $\Pr[piv|\theta] = \binom{n}{\frac{n}{2}}(z_\theta)^{\frac{n}{2}}$. For both kinds of conflicted voters to mix, (σ_a, σ_b) must solve $V_a = V_b = 0$. This implies that $\Pr[piv|\alpha] = \Pr[piv|\beta] = \frac{1}{2r-1}\frac{\varepsilon}{1-\varepsilon}$. The equality of pivot probabilities in the two states implies that either $\gamma_\alpha = \gamma_\beta$ or $\gamma_\alpha = 1 - \gamma_\beta$. It may be readily verified that $\gamma_\alpha - \gamma_\beta > 0$ for all $\sigma_a, \sigma_b \in (0,1)$. Hence, $\gamma_\alpha = 1 - \gamma_\beta$. Finally, $\gamma_\alpha = 1 - \gamma_\beta$ implies that $\sigma_a = 1 - \frac{\rho}{1-\rho}\sigma_b$, which completes the proof. ■

Next, we determine the bounds for which completely mixed voting is an equilibrium. Define $\overline{m}_{CM}$ to be the (unique) value

$$\overline{m}_{CM} \equiv \left\{ m > 0 \; V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} = 0 \right\} \tag{8}$$

Proposition 13 *A completely mixed equilibrium exists iff n is such that $\overline{m}_I < n < \overline{m}_{CM}$. For each such n, there exists exactly one completely mixed equilibrium. Moreover, $\overline{m}_{CM} > \overline{m}_I$.*

Proof. In a completely mixed equilibrium, $V_a = V_b = 0$. From Lemma 6 we know that $\gamma_\alpha = 1 - \gamma_\beta$ and, hence, these equalities reduce to

$$\binom{n}{\frac{n}{2}} \left((2r-1) \left(\gamma_\alpha \left(1 - \gamma_\alpha\right) \right)^{\frac{n}{2}} \right) - \frac{\varepsilon}{1-\varepsilon} = 0 \tag{9}$$

Fact 1: By Lemma 2, the LHS is strictly decreasing in n for fixed γ_α. Fact 2: For fixed n and $\gamma_\alpha > \frac{1}{2}$, the LHS is strictly decreasing in γ_α.

From Lemma 6 we know that, over the range $\sigma_b \in \left(0, \frac{1-\rho}{\rho}\right)$, $\sigma_a = 1 - \frac{\rho}{1-\rho}\sigma_b$. Hence, $\gamma_\alpha \in \left(\gamma_\alpha|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}}, r\right)$, where it is easily verified that $\gamma_\alpha|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} \geq \frac{1}{2}$. Facts 1 and 2 imply that the upper bound on voting body sizes for which a completely mixed equilibrium exists, $\overline{m}_{CM}$, is the value of m solving equation (9) at $\gamma_\alpha = \gamma_\alpha|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}}$. Similarly, the lower bound is the value of m solving equation (9) at $\gamma_\alpha = r$. Notice that this corresponds to $\overline{m}_I$. Facts 1 and 2 also imply that $\overline{m}_I < \overline{m}_{CM}$. Finally, Fact 2 implies that, for all $m \in (\overline{m}_I, \overline{m}_{CM})$, the completely mixed equilibrium is unique. ■

Equilibrium uniqueness turns on the monotonicity of $V_b|_{\sigma_a=0}$ in σ_b. Essentially, if q is high, such that $V_b|_{\sigma_a=0}$ is increasing in σ_b at $\sigma_b = 1$ and $m = \underline{m}_E$, then equilibrium is unique for every n. Formally, q^*, the threshold between high and low q, is defined as the (unique) value

$$q^* \equiv \max \left\{ q \in [0,1] \mid \left. \frac{\partial V_b}{\partial \sigma_b} \right|_{\sigma_a=0,\sigma_b=1,m=\underline{m}_E(q)} = 0 \right\} \tag{10}$$

where $\underline{m}_E(q)$ reflects the dependence of $\underline{m}_E$ on the prevalence of conflict. We now show that

Lemma 7 *q^* exists and is unique. Furthermore, $q_0 < q^* < q_1$, where $q_0 \equiv \frac{1}{2}\frac{2\rho-1}{\rho-r(1-r)}$ and $q_1 \equiv \frac{1}{2}\frac{2\rho-1}{\rho-(1-r)}$.*

Proof. We prove $q_0 < q^* < q_1$ by showing that: 1) for $q \le q_0$ and $m > 0$, $\left.\frac{\partial V_b}{\partial \sigma_b}\right|_{\sigma_a=0,\sigma_b=1} < 0$; 2) for $q \ge q_1$ and $m > 0$, $\left.\frac{\partial V_b}{\partial \sigma_b}\right|_{\sigma_a=0,\sigma_b=1} > 0$. Existence of q^* then follows from continuity of $\left.\frac{\partial V_b}{\partial \sigma_b}\right|_{\sigma_a=0,\sigma_b=1,m=\underline{m}_E(q)}$ in q and the intermediate value theorem, while the max operator in equation (10) guarantees uniqueness.

Notice that

$$\frac{\partial V_b}{\partial \sigma_b} = \frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}\frac{m}{2}(1-q)\rho\left\{r^2(z_\beta)^{\frac{m}{2}-1}\left(1-2\gamma_\beta\right)-(1-r)^2(z_\alpha)^{\frac{m}{2}-1}(1-2\gamma_\alpha)\right\} \tag{11}$$

Hence, $\left.\frac{\partial V_b}{\partial \sigma_b}\right|_{\sigma_a=0,\sigma_b=1}$ takes on the sign of

$$r^2\left(\frac{z_\beta^E}{z_\alpha^E}\right)^{\frac{m}{2}-1}\left(1-2\gamma_\beta^E\right)-(1-r)^2\left(1-2\gamma_\alpha^E\right) \tag{12}$$

By Lemma 8 (below), $z_\alpha^E < z_\beta^E$. Thus, (12) is strictly smaller than

$$r^2\left(1-2\gamma_\beta^E\right)-(1-r)^2\left(1-2\gamma_\alpha^E\right) = (2r-1)\left(2q\left(r^2-r+\rho\right)+1-2\rho\right)$$

which is negative iff $q \le q_0$. Thus, $\left.\frac{\partial V_b}{\partial \sigma_b}\right|_{\sigma_a=0,\sigma_b=1} < 0$ for all m. This establishes 1).

For $q \ge q_1$, $\gamma_\alpha > \frac{1}{2}$ and $\gamma_\beta \le \frac{1}{2}$. Thus, (12) > 0, which establishes 2). ■

Lemma 8 *If $\sigma_a = 0$ and $\sigma_b > \frac{1-\rho}{\rho}$, then $z_\alpha < z_\beta$.*

Proof. It is sufficient to show that $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$. If $\sigma_a = 0$ and $\sigma_b > \frac{1-\rho}{\rho}$, then

$$\gamma_\alpha > qr + (1-q)\left(r\rho + (1-r)(1-\rho)\right) \ge qr + (1-q)\frac{1}{2} > \frac{1}{2}$$

where the first inequality follows from $\sigma_b > \frac{1-\rho}{\rho}$, the second from $\rho \geq \frac{1}{2}$, and the third from $r > \frac{1}{2}$.

If $\gamma_\beta > \frac{1}{2}$, then $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$ follows immediately from the fact that $\gamma_\beta < \gamma_\alpha$.

If $\gamma_\beta \leq \frac{1}{2}$, then $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$ is equivalent to showing that $\gamma_\alpha - \left(1 - \gamma_\beta\right) > 0$. For $\sigma_a = 0$ and $\sigma_b > \frac{1-\rho}{\rho}$,

$$\gamma_\alpha - \left(1 - \gamma_\beta\right) > qr + r\left(1-q\right)\rho + \left(1-q\right)\left(1-\rho\right) + q\left(1-r\right) + \left(1-r\right)\left(1-q\right)\rho - 1 = 0$$

This completes the proof. ■

Proposition 3 implies that, to prove Proposition 4, only partially mixed equilibria remain to be analyzed.

Proposition 14 *Under low conflict (i.e., $q > q^*$), a partially mixed equilibrium exists iff n is such that $\overline{m}_{CM} \leq n < \underline{m}_E$. For each such n, there exists exactly one partially mixed equilibrium. Moreover, $\overline{m}_{CM} < \underline{m}_E$.*

Proof. By Lemma 9 (below), in any partially mixed equilibrium, $\sigma_b \in [\frac{1-\rho}{\rho}, 1)$. Next, we claim that $V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} < 0$ iff $n > \overline{m}_{CM}$. At $\overline{m}_{CM}$, $V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} = 0$ by construction. Moreover, Lemmas 8 and 2 imply that $V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}}$ is strictly decreasing in n. This proves the claim.

We also claim that $V_b|_{\sigma_a=0,\sigma_b=1} > 0$ iff $n < \underline{m}_E$. At $\underline{m}_E$, $V_b|_{\sigma_a=0,\sigma_b=1} = 0$ by construction. Moreover, Lemmas 8 and 2 imply that $V_b|_{\sigma_a=0,\sigma_b=1}$ is strictly decreasing in n. This proves the claim.

From Lemma 11 (below)—which shows that, under low conflict, V_b is strictly increasing in $\sigma_b \in [\frac{1-\rho}{\rho}, 1)$—it then follows that for all $\overline{m}_{CM} \leq n < \underline{m}_E$, there exists a unique value $\sigma_b \in \left(\frac{1-\rho}{\rho}, 1\right)$ such that $V_b|_{\sigma_a=0}\left(\sigma_b\right) = 0$. It is straightforward to verify that, at this value of σ_b, $V_a|_{\sigma_a=0} < 0$. Hence, this constitutes a partially mixed equilibrium.

Finally, we establish that $\overline{m}_{CM} < \underline{m}_E$. At $\overline{m}_{CM}$, $V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} = 0$. Lemma 11 implies that, at $\overline{m}_{CM}$, $V_b|_{\sigma_a=0,\sigma_b=1} > 0$. Moreover, from Lemmas 8 and 2 we know that $V_b|_{\sigma_a=0,\sigma_b=1}$

is strictly decreasing in m. Because, at $\underline{m}_E$, $V_b|_{\sigma_a=0,\sigma_b=1} = 0$, this implies that $\underline{m}_E > \overline{m}_{CM}$.

■

Lemma 9 *In any partially mixed equilibrium,* $\sigma_b \geq \frac{1-\rho}{\rho}$.

Proof. We prove the lemma by showing that $\sigma_b < \frac{1-\rho}{\rho}$ implies $z_\alpha > z_\beta$, which contradicts Lemma 10 (below). Recall that $\gamma_\alpha > \gamma_\beta$. First, we find the value of σ_b that makes $\gamma_\alpha = \frac{1}{2}$. This is readily shown to be $\sigma_b' = \frac{1-2qr-2r(1-q)\rho}{2(1-r)(1-q)\rho}$, while

$$\sigma_b' - \frac{1-\rho}{\rho} = -\frac{(2r-1)(2q(1-\rho)+2\rho-1)}{2(1-r)(1-q)\rho^2} < 0$$

Because γ_α is increasing in σ_b, for $\sigma_b \leq \sigma_b'$, $\gamma_\beta < \gamma_\alpha \leq \frac{1}{2}$ and, hence, $z_\alpha > z_\beta$.

Next, we find the value of σ_b that makes $\gamma_\beta = \frac{1}{2}$. This is readily shown to be $\sigma_b'' = \frac{1-2qr-2r(1-q)\rho}{2(1-r)(1-q)\rho}$, while

$$\sigma_b'' - \frac{1-\rho}{\rho} = \frac{(2r-1)(2q(1-\rho)+2\rho-1)}{2r(1-q)\rho^2} > 0$$

In the region $\sigma_b' < \sigma_b < \sigma_b''$, $\gamma_\beta < \frac{1}{2} < \gamma_\alpha$. As γ_α and γ_β are both strictly increasing in σ_b, $z_\alpha - z_\beta$ is strictly decreasing in σ_b. Now notice that, at $\sigma_b = \frac{1-\rho}{\rho}$, $\gamma_\alpha = 1 - \gamma_\beta$ and, therefore, $z_\alpha = z_\beta$. Hence, we may conclude that for all $\sigma_b < \frac{1-\rho}{\rho}$, $z_\alpha > z_\beta$. This completes the proof.

■

Lemma 10 *In any partially mixed equilibrium,* $z_\alpha \leq z_\beta$.

Proof. In any partially mixed equilibrium, $V_a \leq 0$ and $V_b = 0$. This implies that $V_a + V_b \leq 0$, which may be rewritten as

$$\binom{n}{\frac{n}{2}}\left((z_\alpha)^{\frac{n}{2}} - (z_\beta)^{\frac{n}{2}}\right) \leq 0$$

And this inequality holds iff $z_\alpha \leq z_\beta$. ■

Lemma 11 *For* $q \geq q_1$, $V_b|_{\sigma_a=0}$ *is strictly increasing in* $\sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]$.

Proof. Differentiating $V_b|_{\sigma_a=0}$ with respect to σ_b yields

$$\left.\frac{\partial V_b}{\partial \sigma_b}\right|_{\sigma_a=0} = \binom{n}{\frac{n}{2}} \frac{n}{2} (1-q)\, \rho \left\{ r^2 (z_\beta)^{\frac{n}{2}-1} \left(1 - 2\gamma_\beta\right) - (1-r)^2 (z_\alpha)^{\frac{n}{2}-1} (1 - 2\gamma_\alpha) \right\}$$

which takes the sign of the expression in curly brackets. Notice that γ_β is increasing in σ_b and, for $\sigma_a = 0$, γ_β is decreasing in q. At $q = q_1$, $\gamma_\beta|_{\sigma_a=0,\sigma_b=1} = \frac{1}{2}$. Hence, for $q \geq q_1$ and $\sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]$, we have $\gamma_\beta|_{\sigma_a=0} \leq \frac{1}{2}$. Moreover, for $\sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]$, it can be easily verified that $\gamma_\alpha|_{\sigma_a=0} > \frac{1}{2}$. Finally, together, $\gamma_\beta \leq \frac{1}{2}$ and $\gamma_\alpha > \frac{1}{2}$ imply that the expression in curly brackets is strictly positive. ■

This completes the proof of Proposition 4.

Proof of Proposition 5

Under instrumental and expressive voting, σ_a and σ_b remain constant as n increases. Hence, voting becomes (weakly) more expressive. It remains to show that voting becomes (strictly) more expressive under completely mixed and partially mixed voting, as well as when we move from one equilibrium class to the next.

Since there exists a unique completely mixed equilibrium for every n in the interval $\overline{m}_I < n < \overline{m}_{CM}$, we can define a sequence of completely mixed equilibria, with n running from $\lceil \overline{m}_I \rceil$ to $\lfloor \overline{m}_{CM} \rfloor$. Note that this sequence is fully characterized by the sequence of mixing probabilities $\{\{\sigma_a, \sigma_b\}_n\}_{\lceil \overline{m}_I \rceil < n < \lfloor \overline{m}_{CM} \rfloor}$.

Lemma 12 *In the completely mixed equilibrium sequence, voting becomes more expressive as n increases.*

Proof. Any completely mixed equilibrium is characterized by the unique value $\gamma_\alpha^* > \frac{1}{2}$ that solves equation (9). Lemma 2 implies that the LHS of equation (9) is decreasing in n and, as a consequence, γ_α^* must also be decreasing in n. Using $\sigma_b = \frac{1-\rho}{\rho}(1-\sigma_a)$, it is easily verified that $\frac{d\gamma_\alpha}{d\sigma_a} > 0$. Hence, in any completely mixed equilibrium, σ_a must be decreasing in n, while σ_b is increasing in n. ■

Since there is a unique partially mixed equilibrium for every $\overline{m}_{CM} \leq n < \underline{m}_E$, we can define a sequence of such equilibria, with n running from $\lceil \overline{m}_{CM} \rceil$ to $\lfloor \underline{m}_E \rfloor$. This sequence is fully characterized by the sequence of mixing probabilities $\left\{ \{\sigma_b\}_n \right\}_{\lceil \overline{m}_{CM} \rceil < n < \lfloor \underline{m}_E \rfloor}$.

Lemma 13 *In the partially mixed equilibrium sequence, as n increases, voting becomes more expressive.*

Proof. In a partially mixed equilibrium, σ_b solves $V_b|_{\sigma_a=0}(\sigma_b) = 0$. Lemma 8 together with Lemma 2 imply that, for fixed σ_b, $V_b|_{\sigma_a=0}$ is strictly decreasing in n. Furthermore, we know from Lemma 11 that, for fixed n and $q \geq q_1$, $V_b|_{\sigma_a=0}(\sigma_b)$ is strictly increasing in σ_b. Together, these two facts imply that the equilibrium value of σ_b must be strictly increasing in n. As σ_a remains constant at zero, voting becomes more expressive when n increases. ■

Hence, within each equilibrium class, voting becomes (weakly) more expressive as n increases. Moreover, it is easily verified that voting also becomes more expressive when we move from one equilibrium class to the next. This completes the proof of Proposition 5.

Proof of Proposition 6

Before getting to the heart of the proof, we formally define $\overline{m}_{PM}$ and derive some of its properties. Define $\overline{m}_{PM}$ to be the largest value of m such that the indifference condition for conflicted voters with a b signal still has a solution in σ_b. That is,

$$\overline{m}_{PM} \equiv \max \left\{ m \text{ such that } V_b|_{\sigma_a=0} = 0 \text{ has a solution in } \sigma_b \in \left[\frac{1-\rho}{\rho}, 1 \right] \right\} \tag{13}$$

We denote this solution by $\sigma_{b,\overline{m}_{PM}}$.[11] The next lemma establishes existence and uniqueness of $\overline{m}_{PM}$, and provides sufficient conditions for $\overline{m}_{PM} > \underline{m}_E$ (such that expressive voting overlaps with partially mixed voting).

Lemma 14 *$\overline{m}_{PM}$ exists and is unique. Moreover, for $q \leq q_0$, $\underline{m}_E < \overline{m}_{PM}$.*

[11] While neither $\bar{m}_{PM}$ nor $\sigma_{b,\bar{m}_{PM}}$ admit closed-form solutions, approximations are available. For small ε, $\bar{m}_{PM} \approx \frac{2}{\pi} \left(r \frac{1-\varepsilon}{\varepsilon} \right)^2$ and $\sigma_{b,\bar{m}_{PM}} \approx \frac{\frac{1}{2} - q(1-r)}{r(1-q)\rho} - \frac{1-r}{r}$.

Proof. By Lemma 15 (below), for $q \leq q_0$, the unique σ_b that maximizes $V_b|_{\sigma_a=0}(\sigma_b)$ over the interval $\left[\frac{1-\rho}{\rho}, 1\right]$ is strictly interior. Denote this σ_b by σ_b'. By the envelope theorem,

$$\frac{d}{dm} V_b|_{\sigma_a=0,\sigma_b=\sigma_b'(m)} = \left.\frac{\partial \left(V_b|_{\sigma_a=0}\right)}{\partial m}\right|_{\sigma_b=\sigma_b'(m)}$$

Lemmas 8 and 2 imply that $\left.\frac{\partial(V_b|_{\sigma_a=0})}{\partial m}\right|_{\sigma_b=\sigma_b'(m)}$ and, therefore, $\frac{d}{dm}V_b|_{\sigma_a=0,\sigma_b=\sigma_b'(m)}$ are strictly negative. From here, the proof of existence and uniqueness of $\overline{m}_{PM}$ is analogous to that for $\underline{m}_E$ in the main text.

To prove that $\overline{m}_{PM} > \underline{m}_E$ for $q \leq q_0$, note that, at $m = \underline{m}_E$, $V_b|_{\sigma_a=0,\sigma_b=1} = 0$. By Lemma 15 we know that $\frac{\partial V_b}{\partial \sigma_b}|_{\sigma_a=0,\sigma_b=1} < 0$. Hence, for some σ_b'' strictly smaller than but close to 1, $V_b|_{\sigma_a=0,\sigma_b=\sigma_b''} > 0$. Lemma 2 then implies that there exists an $m > \underline{m}_E$ such that the equation $V_b|_{\sigma_a=0} = 0$ has a solution in $\sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]$. Therefore, $\overline{m}_{PM}$, which is defined as the largest m for which such a solution exists, must also be strictly greater than $\underline{m}_E$. ■

Lemma 15 *For $q \leq q_0$, $V_b|_{\sigma_a=0}(\sigma_b)$ is single-peaked in σ_b on the interval $\left[\frac{1-\rho}{\rho}, 1\right]$. Moreover, the peak is strictly interior.*

Proof. From Lemma 7 we know that $\left.\frac{\partial V_b}{\partial \sigma_b}\right|_{\sigma_a=0,\sigma_b=1} < 0$. From equation (11), we know that $\frac{\partial V_b}{\partial \sigma_b}$ takes the sign of

$$r^2 (z_\beta)^{\frac{n}{2}-1}\left(1-2\gamma_\beta\right) - (1-r)^2 (z_\alpha)^{\frac{n}{2}-1}(1-2\gamma_\alpha)$$

This expression is strictly positive at $\sigma_a = 0$ and $\sigma_b = \frac{1-\rho}{\rho}$, since $\gamma_\alpha > \frac{1}{2}$ and $\gamma_\beta < \frac{1}{2}$, where $\gamma_\beta < \frac{1}{2}$ follows from

$$\begin{aligned} \gamma_\beta|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} &= q(1-r) + (1-q)\left((1-r)\rho + r(1-\rho)\right) \\ &< q(1-r) + (1-q)\frac{1}{2} \leq \frac{1}{2} \end{aligned}$$

Thus, $\left.\frac{\partial V_b}{\partial \sigma_b}\right|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} > 0$.

The intermediate value theorem now implies that there exists at least one $\sigma_b \in \left(\frac{1-\rho}{\rho}, 1\right)$ where $\left.\frac{\partial V_b}{\partial \sigma_b}\right|_{\sigma_a=0} = 0$. We will show that, at any such point, $\left.\frac{d^2 V_b}{(d\sigma_b)^2}\right|_{\sigma_a=0} < 0$. Therefore, $V_b|_{\sigma_a=0}$ is single-peaked on $\sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]$.

First, the FOC can only be satisfied when $\frac{1}{2} < \gamma_\beta < \gamma_\alpha$. Next, the FOC implies that

$$\left(\frac{1-2\gamma_\beta}{1-2\gamma_\alpha}\right)^2 = \left(\frac{1-r}{r}\right)^2 \left(\frac{z_\alpha}{z_\beta}\right)^{n-2} \tag{14}$$

Now notice that $\frac{d^2 V_b}{(d\sigma_b)^2}$ is proportional to

$$\begin{aligned}&\left(\frac{n}{2}-1\right)\left[r^3 (z_\beta)^{\frac{n}{2}-2}\left(1-2\gamma_\beta\right)^2 - (1-r)^3 (z_\alpha)^{\frac{n}{2}-2}(1-2\gamma_\alpha)^2\right] \\ &+2\left\{(1-r)^3 (z_\alpha)^{\frac{n}{2}-1} - r^3 (z_\beta)^{\frac{n}{2}-1}\right\}\end{aligned}$$

Since $\frac{1}{2} < \gamma_\beta < \gamma_\alpha$, the term in curly brackets is negative. For the term in square brackets to be negative, we need to show that

$$\frac{r^3}{(1-r)^3}\frac{(z_\beta)^{\frac{n}{2}-2}}{(z_\alpha)^{\frac{n}{2}-2}}\frac{\left(1-2\gamma_\beta\right)^2}{(1-2\gamma_\alpha)^2} < 1$$

Substituting in equation (14), we have

$$\left(\frac{z_\alpha}{z_\beta}\right)^{\frac{n}{2}}\frac{1-r}{r} < 1$$

where the required inequality holds because $\frac{1}{2} < \gamma_\beta < \gamma_\alpha$ and $r > \frac{1}{2}$. ■

We are now in a position to prove Proposition 6. The proofs of parts (1), (2), and (4) are identical to the proofs of Proposition 1, Lemma 13, and Proposition 2, respectively. It remains to show that: 1) Low partially mixed voting is an equilibrium iff $\overline{m}_{CM} \leq n < \overline{m}_{PM}$. 2) High partially mixed voting is an equilibrium iff $\underline{m}_E \leq n < \overline{m}_{PM}$. 3) If a low, respectively, high partially mixed equilibrium exists, it is unique.

First, the proof of Lemma 14 implies that $\overline{m}_{PM}$ constitutes the upper bound on partially mixed voting. Note that Lemma 9 holds independently of q. Thus, we may apply the same reasoning as in the proof of Lemma 14 to conclude that $\overline{m}_{CM}$ is the lower bound for low partially mixed voting. The argument as to why $V_b|_{\sigma_a=0,\sigma_b=1} < 0$ iff $n > \underline{m}_E$ is unchanged from the low conflict case. Thus, we may conclude that $\underline{m}_E$ is the lower bound for high partially mixed voting. Finally, uniqueness follows from Lemma 15.

This completes the proof of Proposition 6.

Proof of Proposition 8

The second part of the proposition follows immediately from the law of large numbers and the fact that, for $q < q_1$, $1 - \gamma_\beta^E < \frac{1}{2} < \gamma_\alpha^E$.

To prove the first part of the proposition, note that adding two voters to a voting body of $n-1$ voters affects the outcome only if, after $n-1$ votes, either: 1) the correct choice is lagging by one vote and the next two votes are "successes," or 2) the correct choice is leading by one vote and the next two votes are "failures." This implies that

$$\begin{aligned} S(n+1|\alpha) - S(n-1|\alpha) &= \binom{n-1}{\frac{n}{2}-1} (z_\alpha)^{\frac{n}{2}} (2\gamma_\alpha - 1) \text{ and} \\ S(n+1|\beta) - S(n-1|\beta) &= -\binom{n-1}{\frac{n}{2}-1} (z_\beta)^{\frac{n}{2}} \left(2\gamma_\beta - 1\right) \end{aligned}$$

Hence,

$$S(n+1) - S(n-1) = \frac{1}{2}\binom{n-1}{\frac{n}{2}-1}\left((z_\alpha)^{\frac{n}{2}}(2\gamma_\alpha - 1) - (z_\beta)^{\frac{n}{2}}\left(2\gamma_\beta - 1\right)\right)$$

For n sufficiently large, the sign of this expression is negative iff

$$\left(\frac{z_\alpha^E}{z_\beta^E}\right)^{\frac{n}{2}} < \frac{\gamma_\beta^E - \frac{1}{2}}{\gamma_\alpha^E - \frac{1}{2}}$$

Lemma 8 implies that the LHS is decreasing in n and goes to zero in the limit. The RHS is a positive constant. Thus, for sufficiently large n, $S(n+1)$ is decreasing.

Proof of Proposition 9

In a completely mixed equilibrium, $S(n+1) = \frac{B\left(\gamma, \frac{n}{2}+1, \frac{n}{2}+1\right)}{B\left(\frac{n}{2}+1, \frac{n}{2}+1\right)}$, where $\gamma \equiv \gamma_a = 1 - \gamma_\beta$. The Proposition follows immediately from the following lemma, which shows that, if the probability of being pivotal remains constant as n increases, then accuracy must fall.

Lemma 16 *Let* $\frac{1}{2} < \gamma - \delta < \gamma < 1$. *If*

$$\frac{\Gamma(n-1)}{\Gamma^2\left(\frac{n}{2}\right)}(\gamma(1-\gamma))^{\frac{n}{2}-1} = \frac{\Gamma(n+1)}{\Gamma^2\left(\frac{n}{2}+1\right)}((\gamma-\delta)(1-(\gamma-\delta)))^{\frac{n}{2}} \tag{15}$$

Then

$$\frac{B\left(\gamma, \frac{n}{2}, \frac{n}{2}\right)}{B\left(\frac{n}{2}, \frac{n}{2}\right)} - \frac{B\left(\gamma - \delta, \frac{n}{2}+1, \frac{n}{2}+1\right)}{B\left(\frac{n}{2}+1, \frac{n}{2}+1\right)} > 0$$

Proof. Define the gap between γ and $\frac{1}{2}$ to be $g \equiv \gamma - \frac{1}{2}$. Then equation (15) can be rewritten as

$$\frac{\Gamma(n-1)}{\Gamma^2\left(\frac{n}{2}\right)}\left(\frac{1}{4}\right)^{\frac{n}{2}-1}\left(1-4g^2\right)^{\frac{n}{2}-1} = \frac{\Gamma(n-1)\,n\,(n-1)}{\Gamma^2\left(\frac{n}{2}\right)\left(\frac{n}{2}\right)^2}\left(\frac{1}{4}\right)^{\frac{n}{2}}\left(1-4(g-\delta)^2\right)^{\frac{n}{2}}$$

Simplifying yields the equality

$$\frac{n}{n-1}\left(1-4g^2\right)^{\frac{n}{2}-1} = \left(1-4(g-\delta)^2\right)^{\frac{n}{2}} \tag{16}$$

Next, note that

$$\frac{1}{B\left(\frac{n}{2}+1, \frac{n}{2}+1\right)} = \frac{2(n+1)}{\frac{n}{2}}\frac{1}{B\left(\frac{n}{2}, \frac{n}{2}\right)}$$

Thus, using the integral representation of the incomplete Beta function, we need only show that

$$\int_0^{\gamma}(t(1-t))^{\frac{n}{2}-1}\,dt - \frac{2(n+1)}{\frac{n}{2}}\int_0^{\gamma-\delta}(t(1-t))^{\frac{n}{2}}\,dt > 0$$

Defining $u = t - \frac{1}{2}$, we may rewrite the LHS as

$$\left(\frac{1}{4}\right)^{\frac{n}{2}-1}\left\{\int_{-\frac{1}{2}}^{g}\left(1-4u^2\right)^{\frac{n}{2}-1}du - \frac{n+1}{n}\int_{-\frac{1}{2}}^{g-\delta}\left(1-4u^2\right)^{\frac{n}{2}}du\right\}$$

Thus, it suffices to show that the term in curly brackets is strictly positive. This term may be rewritten as

$$\begin{aligned}
& \int_{-\frac{1}{2}}^{g-\delta}\left(1-4u^2\right)^{\frac{n}{2}-1}\left(1-\frac{n+1}{n}\left(1-4u^2\right)\right)du + \int_{g-\delta}^{g}\left(1-4u^2\right)^{\frac{n}{2}-1}du \\
= & \int_{-\frac{1}{2}}^{g-\delta}4u^2\left(1-4u^2\right)^{\frac{n}{2}-1}du - \frac{1}{n}\int_{-\frac{1}{2}}^{g-\delta}\left(1-4u^2\right)^{\frac{n}{2}}du + \int_{g-\delta}^{g}\left(1-4u^2\right)^{\frac{n}{2}-1}du
\end{aligned}$$

Now, integrating the first term of this expression by parts, we obtain

$$\begin{aligned}
& -\frac{1}{n}(g-\delta)\left(1-4(g-\delta)^2\right)^{\frac{n}{2}} + \frac{1}{n}\int_{-\frac{1}{2}}^{g-\delta}\left(1-4u^2\right)^{\frac{n}{2}}du \\
& -\frac{1}{n}\int_{-\frac{1}{2}}^{g-\delta}\left(1-4u^2\right)^{\frac{n}{2}}du + \int_{g-\delta}^{g}\left(1-4u^2\right)^{\frac{n}{2}-1}du \\
= & -\frac{1}{n}(g-\delta)\left(1-4(g-\delta)^2\right)^{\frac{n}{2}} + \int_{g-\delta}^{g}\left(1-4u^2\right)^{\frac{n}{2}-1}du
\end{aligned}$$

Recall that, for all u in the support of the second term, $\frac{u}{g} < 1$. Hence,

$$\begin{aligned} & -\frac{1}{n}(g-\delta)\left(1-4(g-\delta)^2\right)^{\frac{n}{2}}+\int_{g-\delta}^{g}\left(1-4u^2\right)^{\frac{n}{2}-1}du \\ > \; & -\frac{1}{n}(g-\delta)\left(1-4(g-\delta)^2\right)^{\frac{n}{2}}+\frac{1}{g}\int_{g-\delta}^{g}u\left(1-4u^2\right)^{\frac{n}{2}-1} \\ = \; & -\frac{1}{n}(g-\delta)\left(1-4(g-\delta)^2\right)^{\frac{n}{2}}-\frac{1}{g}\frac{1}{4n}\left(1-4g^2\right)^{\frac{n}{2}}+\frac{1}{g}\frac{1}{4n}\left(1-4(g-\delta)^2\right)^{\frac{n}{2}} \end{aligned} \quad (17)$$

Using equation (16) to substitute for $\left(1-4(g-\delta)^2\right)^{\frac{n}{2}}$, equation (17) reduces to

$$\begin{aligned} & -\frac{1}{n}(g-\delta)\frac{n}{n-1}\left(1-4g^2\right)^{\frac{n}{2}-1}-\frac{1}{g}\frac{1}{4n}\left(1-4g^2\right)^{\frac{n}{2}}+\frac{1}{g}\frac{1}{4n}\frac{n}{n-1}\left(1-4g^2\right)^{\frac{n}{2}-1} \\ = \; & \frac{1}{g}\left(1-4g^2\right)^{\frac{n}{2}-1}\left[-(g-\delta)g\frac{1}{n-1}-\frac{1}{4n}\left(1-4g^2\right)+\frac{1}{4}\frac{1}{n-1}\right] \end{aligned}$$

It suffices to show that the term in square brackets is positive. Rewriting this expression, we have

$$\frac{1}{4}\left(\left(\frac{1}{n-1}-\frac{1}{n}\right)\left(1-4g^2\right)+\frac{1}{n-1}4g\delta\right)$$

which is strictly positive since $1-4g^2>0$ and $\delta>0$. ■

Proof of Proposition 10

To prove the proposition, the following lemma is useful. Denote by $S\left(\gamma_\alpha,\gamma_\beta\right)$ the accuracy of a fixed size voting body when the probability of a vote for A in state α is equal to γ_α, while the probability of a vote for A in state β is equal to γ_β.

Lemma 17 *Fix $\gamma_\alpha \geq \gamma_\beta$ and let $0 \leq \delta < 1-\gamma_\alpha$.*
If $(\gamma_\alpha+\delta)(1-(\gamma_\alpha+\delta)) < \left(1-\left(\gamma_\beta+\delta\right)\right)\left(\gamma_\beta+\delta\right)$, then $\frac{d}{d\delta}S\left(\gamma_\alpha+\delta,\gamma_\beta+\delta\right)<0$

Proof. Using the Beta function representation of accuracy, we have

$$\frac{d}{d\delta}S\left(\gamma_\alpha+\delta,\gamma_\beta+\delta\right)=\frac{1}{2}\frac{\left((\gamma_\alpha+\delta)(1-(\gamma_\alpha+\delta))\right)^{\frac{n}{2}}-\left(\left(1-\left(\gamma_\beta+\delta\right)\right)\left(\gamma_\beta+\delta\right)\right)^{\frac{n}{2}}}{\int_0^1\left(t(1-t)\right)^{\frac{n}{2}}dt}$$

which is strictly negative, since $(\gamma_\alpha+\delta)(1-(\gamma_\alpha+\delta)) < \left(1-\left(\gamma_\beta+\delta\right)\right)\left(\gamma_\beta+\delta\right)$. ■

We divide the remainder of the proof of Proposition 10 into four parts.

1) $S_{HPM} > S_E$: Note that $\gamma_\alpha^{HPM} < \gamma_\alpha^E$, while $\gamma_\beta^{HPM} < \gamma_\beta^E$. Next, note that

$$\begin{aligned}\gamma_\alpha^E - \gamma_\alpha^{HPM} &= (1-r)(1-q)\rho(1-\sigma_b) \\ &< r(1-q)\rho(1-\sigma_b) = \gamma_\beta^E - \gamma_\beta^{HPM}\end{aligned}$$

Lemma 8 implies that, for all $0 < \delta < 1 - \gamma_\alpha^{HPM}$, $\left(\gamma_\alpha^{HPM} + \delta\right)\left(1 - \left(\gamma_\alpha^{HPM} + \delta\right)\right) <$ $\left(1 - \left(\gamma_\beta^{HPM} + \delta\right)\right)\left(\gamma_\beta^{HPM} + \delta\right)$. Now define $\Delta = \gamma_\alpha^E - \gamma_\alpha^{HPM} > 0$. Lemma 17 implies that

$$S_{HPM} = S\left(\gamma_\alpha^{HPM}, \gamma_\beta^{HPM}\right) > S\left(\gamma_\alpha^{HPM} + \Delta, \gamma_\beta^{HPM} + \Delta\right) > S^E$$

2) $S_I > S_{HPM}$: Note that $\gamma_\alpha^I < \gamma_\alpha^{HPM}$, while $\gamma_\beta^I < \gamma_\beta^{HPM}$. Next, note that

$$\begin{aligned}\gamma_\alpha^{HPM} - \gamma_\alpha^I &= (1-q)((1-r)\rho\sigma_b - r(1-\rho)) \\ &< (1-q)(r\rho\sigma_b - (1-r)(1-\rho)) = \gamma_\beta^{HPM} - \gamma_\beta^I\end{aligned}$$

Because $\gamma_\alpha^I\left(1-\gamma_\alpha^I\right) = \gamma_\beta^I\left(1-\gamma_\beta^I\right)$, we have $\left(\gamma_\alpha^I + \delta\right)\left(1 - \left(\gamma_\alpha^I + \delta\right)\right) < \left(1 - \left(\gamma_\beta^I + \delta\right)\right)\left(\gamma_\beta^I + \delta\right)$, for all $\delta \leq 1 - \gamma_\alpha^{HPM}$. The remainder of the proof is analogous to 1).

3) $S_{CM} > S_{HPM}$: Since $\sigma_a^{CM} = 1 - \frac{\rho}{1-\rho}\sigma_b^{CM}$ and $\sigma_b^{CM} < \frac{1-\rho}{\rho} < \sigma_b^{HPM}$, we have $\gamma_\beta^{CM} <$ γ_β^{HPM}. If $\gamma_\alpha^{CM} > \gamma_\alpha^{HPM}$, then accuracy deteriorates in both states and, hence, $S_{CM} > S_{HPM}$. Else, note that

$$\begin{aligned}\gamma_\alpha^{HPM} - \gamma_\alpha^{CM} - \left(\gamma_\beta^{HPM} - \gamma_\beta^{CM}\right) &= (1-q)(2r-1)\left(2\rho\sigma_b^{CM} - \rho\sigma_b^{HPM} - (1-\rho)\right) \\ &< (1-q)(2r-1)(2(1-\rho) - 2(1-\rho)) = 0\end{aligned}$$

Finally, since $\gamma_\alpha^{CM}\left(1-\gamma_\alpha^{CM}\right) = \gamma_\beta^{CM}\left(1-\gamma_\beta^{CM}\right)$, using arguments analogous to those in 2), $S_{CM} > S_{HPM}$.

4) $S_{LPM} > S_{HPM}$: Note that $\gamma_\alpha^{LPM} < \gamma_\alpha^{HPM}$, while $\gamma_\beta^{LPM} < \gamma_\beta^{HPM}$. Next, note that

$$\begin{aligned}\gamma_\alpha^{HPM} - \gamma_\alpha^{LPM} &= (1-r)(1-q)\rho(\sigma_{b,HPM} - \sigma_{b,LPM}) \\ &< r(1-q)\rho(\sigma_{b,HPM} - \sigma_{b,LPM}) = \gamma_\beta^{HPM} - \gamma_\beta^{LPM}\end{aligned}$$

The remainder of the proof is analogous to 1).

This completes the proof of Proposition 10.

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